MTE-12

ASSIGNMENT BOOKLET

LINEAR PROGRAMMING

Valid from 1st Jan, 2025 to 31st Dec, 2025



School of Sciences Indira Gandhi National Open University Maidan Garhi, New Delhi-110068

(2025)

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, which would consist of one tutor-marked assignment for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

| | ROLL NO.: |
|-----------------|--------------|
| | NAME: |
| | ADDRESS: |
| | |
| | |
| COURSE CODE: | |
| COURSE TITLE: | |
| ASSIGNMENT NO.: | · |
| STUDY CENTRE: | DATE: |

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) This assignment is **valid from 1**st **Jan, 2025 to 31**st **Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.
- 7) You cannot fill the examination form for this course until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment (To be done after studying all the blocks)

Course Code: MTE-12 Assignment Code: MTE-12/TMA/2025 Maximum Marks: 100

- 1. State which of the following statements are true and which are false. Give reasons for your answer with a short proof or a counter example.
 - a) The intersection of finite number of convex sets is not convex.
 - b) If value of the 2×2 matrix game $\begin{bmatrix} 1 & 2 \\ p & 4 \end{bmatrix}$ is 4, then $p \ge 4$.
 - c) If 10 is added to each of the entries of the cost matrix of a 3×3 assignment problem, then the total cost of an optimal assignment for the changed cost matrix will increase by 10.
 - d) For maximization LP model, the simplex method is terminated when all values $c_i z_i \ge 0$.
 - e) The dummy source or destination in a transportation problem is added to prevent solution from becoming degenerate.
- a) Reduce the following two person zero sum game to 2×2 game using principle of dominance. And hence solve the game.

b) Obtain the dual of the following primal LP problem:

Maximize $z = x_1 - 2x_2 + 3x_3$ Subject to $-2x_1 + x_2 + 3x_3 = 2$ $2x_1 + 3x_2 + 4x_3 = 1$ $x_1, x_2x_3 \ge 0$

a) A company makes two kinds of leather belts. Belts A is high quality belt and belt B is of lower quality. The respective profits on A and B are ₹ 4 and ₹ 3 per belt. The production of each type A requires twice as much time as a belt. The production of each type B, and if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A

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and *B* combined). Belt *A* require a fancy buckle and only 400 buckles per day are available. There are only 700 buckles a day available for belt *B*. What should be the daily production of each type of belt? Formulate this problem as an LP model and solve it by the graphical method.

b) Find the initial basic feasible solution of the following transportation problem using North-West Corner method.

| 90 | 90 | 100 | 110 | 200 |
|----|-----|-----|-----|-----|
| 50 | 70 | 130 | 85 | 50 |
| 75 | 100 | 100 | 30 | J |

4. a) Two breakfast food manufacturers *ABC* and *XYZ* are competing for an increased market share. The pay-off matrix, shown in the following table, describes the increase in market share for *ABC* and decrease in market share of *XYZ*. Determine optimal strategies for both the manufacturers and the value of the game.

b) Find all the basic feasible solutions of the following system of linear equations:

$$2x_1 + x_2 - x_3 + 2x_4 = 2$$

$$3x_1 + 2x_2 + x_3 + 4x_4 = 3$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Check if any of them is degenerate solution. Justify your answer.

5. a) A department has five employees with five jobs to be performed. The time (in hours) each employee will take to perform each job is given in the following matrix. How should the jobs be allocated, one per employee, so as to minimize the total man hours?

| | | Employees | | | | |
|------|---|-----------|----|--------------------------|----|----|
| | | Ι | Π | III | IV | V |
| | A | 10 | 5 | 13 | 15 | 16 |
| Jobs | В | 3 | 9 | 18 | 13 | 6 |
| JODS | С | 10 | 7 | 2 | 2 | 2 |
| | D | 7 | 11 | 9 | 7 | 12 |
| | Ε | 7 | 9 | 13 18 2 9 10 | 4 | 12 |

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b) For the following pay-off matrix, transform the zero-sum game into an equivalent linear programming problem:

Player *B*
Player *A*
$$\begin{array}{c|ccc} B_1 & B_2 & B_3 \\ \hline A_1 & 1 & -1 & 3 \\ A_1 & 3 & 5 & -3 \\ A_3 & 6 & 2 & -2 \end{array}$$

6. a) Using matrix – minima method, find the initial basic feasible solution of the following transportation problem:

| 4 | 6 | 8 | 8 | 40 |
|----|----|----|----|----|
| 6 | 8 | 6 | 7 | 60 |
| 5 | 7 | 6 | 8 | 50 |
| 20 | 30 | 50 | 50 | |

Hence find the optimal solution.

b) Check whether the following set is convex:

$$S = \{(x, y) : x^2 + y^2 \le 1, y^2 \ge x\}$$

7. a) For what value of k are the following vectors linearly independent? (4)

| $\begin{bmatrix} 1 \end{bmatrix}$ | | 1 | | k | |
|-----------------------------------|---|---|---|---|--|
| 2 | , | k | , | 0 | |
| 1 | | 1 | | 1 | |

b) Solve the following LP problem using simplex method:

Maximize $z = 6x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \le 30$ $3x_1 + 2x_2 \le 24$ $x_1 + x_2 \ge 3$ $x_1, x_2 \ge 0$

8. a) Write the LPP formulation of the following transportation problem:

| | | Destination | | | Supply |
|-------------|-------|-------------|-------|-------|--------|
| | | D_1 | D_2 | D_3 | |
| | O_1 | 10 | 18 | 12 | 200 |
| Source | O_2 | 15 | 17 | 9 | 300 |
| | O_3 | 13 | 15 | 7 | 500 |
| Requirement | | 400 | 200 | 400 | |

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b) Solve the following assignment problem for profit maximization:

| | A | В | С | D |
|-----|----|----|----|----|
| Ι | 14 | 18 | 11 | 26 |
| II | 17 | 23 | 20 | 27 |
| III | 28 | 31 | 26 | 30 |
| IV | 23 | 30 | 25 | 28 |

9. a) Write the LPP formulation of the following assignment problem:

| | Machines | | | |
|------------------------|-------------------|----|----------|--|
| | M_1 M_2 M_3 | | | |
| ${J}_1$ | [18 | 16 | 12 | |
| J_1 Jobs J_2 J_3 | 10 | 7 | 10 18 | |
| J_3 | 14 | 8 | 18 | |

b) Solve the following game graphically:

Player B
Player A
$$\begin{bmatrix} 3 & 7 \\ 5 & 2 \\ 1 & 4 \end{bmatrix}$$

10. a) Solve the following LPP graphically:

Maximize:

$$z = 10x_1 + 10x_2$$

subject to the constraints:

$$4x_1 + 3x_2 \le 12 6x_1 + 18x_2 \le 36 x_1, x_2 \ge 0.$$

b) Find all values of *k* for which the vectors:

| $\begin{bmatrix} 1 \end{bmatrix}$ | | 0 | | 2 |
|-----------------------------------|---|----|-----|----------------------|
| 0 | , | -1 | and | -k |
| 1 | | 1 | | $\lfloor 2k \rfloor$ |

are linearly independent.

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