MTE-07

# ASSIGNMENT BOOKLET

# ADVANCED CALCULUS

**Valid from 1st January, 2025 to 31st December, 2025**

**School of Sciences**
**Indira Gandhi National Open University**
**Maidan Garhi, New Delhi-110068**
**(2025)**

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

**Instructions for Formating Your Assignments**

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ……………………………..…

STUDY CENTRE: …………………………..….. DATE: ……………………………….……………...

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**PLEASE FOLLOW THE FORMAT ABOVE STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

**We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

# ASSIGNMENT

**Course Code: MTE-07**
**Assignment Code: MTE-07/TMA/2025**
**Maximum Marks: 100**

1. State whether the following statements are true or false. Give reasons for your answers. (10)

   (i) $\lim_{x\to 0} \dfrac{x^2 \sin \frac{1}{x}}{\sin x} = 1$

   (ii) A real-valued function of three variables which is continuous everywhere is differentiable.

   (iii) The function $F : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $F(x, y) = (y+2, x+y)$, is locally invertible at any $(x, y) \in \mathbb{R}^2$.

   (iv) $f : [-1,1]\times[-2,2] \to \mathbb{R}$, defined by

   $$f(x, y) = \begin{cases} x, & \text{if } y \text{ is rational} \\ 0, & \text{if } y \text{ is not rational} \end{cases}$$

   is integrable.

   (v) The function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by $f(x, y) = 1 - y^2 + x^2$, has an extremum at $(0,0)$.

2) (a) Find the following limits: (4)

   (i) $\lim_{x\to +\infty} \left(\dfrac{x^2}{8x^2 - 3}\right)^{1/3}$

   (ii) $\lim_{x\to 0^+} (\sin x)^{\sin x}$

   (b) Find the third Taylor polynomial of the function $f\ (x, y) = 1 + 5xy + 3^2 y$ at $(1,2)$. (3)

   (c) Using only the definitions, find $f_{xy}(0,0)$ and $f_{yx}(0,0)$, if they exists, for the function

   $$f(x, y) = \begin{cases} \dfrac{x^2 y}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0,0) \\ 0, & \text{otherwise} \end{cases}$$ (3)

3) (a) Let the function $f$ be defined by

   $$f(x, y) = \begin{cases} \dfrac{3x^2 y^4}{x^4 + y^8}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$$

   Show that $f$ has directional derivatives in all directions at $(0,0)$. (3)

(b) Let $x = e^r \cos\theta$, $y = e^r \sin\theta$ and $f$ be a continuously differentiable function of $x$ and $y$, whose partial derivatives are also continuously differentiable. Show that

$$\frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial \theta^2} = (x^2 + y^2)\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) \quad (5)$$

(c) Let $a = (1,2,3)$, $b = (-5,3,-2)$, $c = (2,-4,1)$ be three points in $\mathbb{R}^3$. Find $|2b - a + 3c|$. (2)

4. (a) Find the centre of gravity of a thin sheet with density $\delta(x, y) = y$, bounded by the curves $y = 4x^2$ and $x = 4$. (5)

(b) Find the mass of the solid bounded by $z = 1$ and $z = x^2 + y^2$, the density function being $\delta(x, y, z) = |x|$. (5)

5. (a) State Green's theorem, and apply it to evaluate

$$\int_C (3x^2 - 4y)\,dx - (2x + y^3)\,dy,$$

Where $C$ is the ellipse $4x^2 + 9y^2 = 36$. (4)

(b) Find the extreme values of the function

$f(x, y) = x^2 + y$ on the surface $x^2 + 2y^2 = 1$. (6)

6. (a) State a necessary condition for the functional dependence of two differentiable functions $f$ and $g$ on an open subset $D$ of $\mathbb{R}^2$. Verify this theorem for the functions $f$ and $g$, defined by

$$f(x, y) = \frac{y - x}{y + x}, \; g(x, y) = \frac{x}{y}. \quad (4)$$

(b) Using the Implicit Function Theorem, show that there exists a unique differentiable function $g$ in a neighbourhood of 1 such that $g(1) = 2$ and $F(g(y), y) = 0$ in a neighbourhood of $(2,1)$, where

$$F(x, y) = x^5 + y^5 - 16xy^3 - 1 = 0$$

defines the function $F$. Also find $g'(y)$. (3)

(c) Check the local inevitability of the function $f$ defined by $f(x, y) = (x^2 - y^2, 2xy)$ at $(1,-1)$. Find a domain for the function $f$ in which $f$ is invertible. (3)

7. (a) Check the continuity and differentiability of the function at $(0,0)$ where

$$f(x, y) = \begin{cases} \dfrac{2x^3 y}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

(b) Find the domain and range of the function $f$, defined by $f(x, y) = \frac{2xy}{x^2 + y^2}$. Also find two level curves of this function. Give a rough sketch of them. (4)

8. (a) Evaluate $\int_C (2x^2 + 3y^2)\, dx$, where $C$ is the curve given by

$x(t) = at^2, y(t) = 2at, 0 \le t \le 1.$ (5)

(b) Use double integration of find the volume of the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1.$$ (5)

9. (a) Find the values of $a$ and $b$, if

$$\lim_{x \to \infty} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$$ (5)

(b) Suppose $S$ and $C$ are subsets of $\mathbb{R}^3$. $S$ is the unit open sphere with centre at the origin and $C$ is the open cube $= \{P(x, y, z) \mid -1 < x < 1, -1 < y < 1, -1 < z < 1\}$.

Which of the following is true. Justify your answer. (3)

(i) $S \subset C$

(ii) $C \subset S$

(c) Identify the level curves of the following functions: (2)

(i) $\sqrt{x^2 + y^2}$

(ii) $\sqrt{4 - x^2 - y^2}$

(iii) $x - y$

(iv) $y / x$

10. (a) Does the function (4)

$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$, $x \ne 0, y \ne 0$ satisfy the requirement of Schwarz's theorem at $(1,1)$? Justify your answer.

(b) Locate and classify the stationary points of the following: (6)

(i) $f(x, y) = 4xy + x^4 - y^4$

(ii) $f(x, y) = xy + \frac{2}{x} + \frac{4}{y}, x > 0,\ y > 0$