MTE-02

# ASSIGNMENT BOOKLET

# LINEAR ALGEBRA

(Valid from 1st January, 2025 to 31st December, 2025)

School of Sciences
Indira Gandhi National Open Universit
Maidan Garhi, New Delhi
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

**Instructions for Formatting Your Assignments**

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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**ROLL NO. :** . . . . . . . . . . . . . . . . . . . . . . . . .

**NAME :** . . . . . . . . . . . . . . . . . . . . . . . . .

**ADDRESS :** . . . . . . . . . . . . . . . . . . . . . . . . .

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**COURSE CODE :** . . . . . . . . . . . . . . . . . . . . . . . . .                . . . . . . . . . . . . . . . . . . . . . . . . .

**COURSE TITLE :** . . . . . . . . . . . . . . . . . . . . . . . .

**STUDY CENTRE :** . . . . . . . . . . . . . . . . . . . . . . . .                **DATE** . . . . . . . . . . . . . . . . . . . . . . . . .

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**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

7) This assignment is valid only up to $31^{st}$ December, 2025. If you fail in this assignment or fail to submit it by $31^{st}$ December, 2025, then you need to get the assignment for the year 2026 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9) **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

# Assignment

**Course Code: MTE-02**
**Assignment Code:MTE-02/TMA/2025**
**Maximum Marks: 100**

1) Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample.

i) The function $f\colon \mathbf{R} \to \mathbf{R}$ defined by $f(x) = \cos x$ is 1-1.

ii) The operation $*$ defined by $x * y = \log(xy)$ is a binary operation on $S$, where $S$ is the set $\{x \in \mathbf{R} | x > 0\}$.

iii) The set $\{(x_1.x_2,\ldots,x_n)\,|x_1,x_2,\ldots,x_n \in \mathbf{R}, x_1 = 2x_2+3\}$ is a subspace of $\mathbf{R}^n$.

iv) There is no $7 \times 5$ matrix of rank 6.

v) If $V$ and $V'$ are vector spaces and $T\colon V \to V'$ is a linear transformation, then whenever $u_1,u_2,\ldots,u_k$ are linearly independent, $Tu_1$, $Tu_2$, …, $Tu_k$ are also linearly independent.

vi) If $V$ is a vector space and $T\colon V \to V$ is a linear operator with $\det(T) = 0$, then $T$ is not diagonalisable.

vii) The degree of the minimal polynomial of a $3 \times 3$ matrix is at most 2.

viii) For any $2 \times 2$ matrix $A$, $\text{Adj}\,(A^t) = (\text{Adj(A)})^t$.

ix) The only matrix which is both symmetric and skew-symmetric is the zero matrix.

x) There is no co-ordinate transformation that transforms the quadratic form $x^2+y^2+z^2$ to the quadratic form $xz+yz$. (20)

2) a) Consider the funtion $f\colon \mathbf{R} \setminus \{-1\} \to \mathbf{R}$ defined by $f(x) = \frac{2x+1}{x+1}$.

i) Check that $f(x)$ is well defined and $1-1$. (3)

ii) Check that $f(x) \neq 2$ for any $x \in \mathbf{R}$. (2)

iii) Check that $g\colon \mathbf{R} \setminus \{2\} \to \mathbf{R}$ given by $g(x) = \frac{x-1}{2-x}$ is well defined and $1-1$. Further, check that $g(x) \neq -1$ for any $x \in \mathbf{R}$. (4)

iv) Check that $(f \circ g)(x) = x$ for $x \in \mathbf{R} \setminus \{2\}$ and $(g \circ f)(x) = x$ for $x \in \mathbf{R} \setminus \{-1\}$. (4)

b) Find the direction cosines of the perpendicular from the origin to the plane $\mathbf{r} \cdot (6\mathbf{i}+4\mathbf{j}+2\sqrt{3}\mathbf{k})+2 = \mathbf{0}$. (2)

3) Let $V$ be the set of all functions that are twice differentiable in $\mathbf{R}$ and

$$S = \{\cos x, \sin x, x\cos x, x\sin x\}.$$

a) Check that $S$ is a linearly independent set over $\mathbf{R}$. (**Hint:** Consider the equation

$$a_0\cos x + a_1\sin x + a_2 x\cos x + a_3 x\sin x.$$

Put $x = 0$, $\pi$, $\frac{\pi}{2}$, $\frac{\pi}{4}$, etc. and solve for $a_i$. ) (5)

b) Let $W = [S]$ and let $T\colon V \to V$ be the function defined by

$$T(f(x)) = \frac{d^2}{dx^2}(f(x)) + 2\frac{d}{dx}(f(x)).$$

Check that $T$ is a linear transformation on $V$. (3)

c) Check that $T(W) \subset W$. (7)

d) Write down the matrix of $T$ on $W$ w.r.t the basis $S$. (2)

e) Is the matrix of the linear operator $T$ non-singular? Justify your answer. (3)

4) a) Show that, if $A$ is any $n \times n$ matrix with real entries, then there is a $n \times n$ symmetric matrix $S$ and a $n \times n$ skew symmetric matrix $S'$ such that $A = S + S'$. (3)

b) Find the solutions to the following system of equations by reducing the corresponding augmented matrix to row-reduced echelon form. (5)

$$2a + 3b + 4c + d = 8$$
$$a + 2b + 2c + 2d = 3$$
$$a - b + c + 3d = 3$$

5) a) For the following matrices, check whether there exists an invertible matrix $P$ such that $P^{-1}AP$ is diagonal. When such a $P$ exists, find $P$. (11)

i) $A = \begin{bmatrix} 0 & 1 & -3 \\ 2 & -1 & 6 \\ 1 & -1 & 4 \end{bmatrix}$ ii) $B = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

b) Find the inverse of the matrix $B$ in part a) by using Cayley-Hamilton theorem. (3)

c) Using the fact that $\det(AB) = \det(A)\det(B)$ for any two matrices $A$ and $B$, prove the identity

$$\left(a^2 + b^2\right)\left(c^2 + d^2\right) = (ac - bd)^2 + (ad + bc)^2$$ (3)

6) a) Find the values of $a$, $b \in \mathbf{C}$ for which the matrix

$$\begin{bmatrix} 1 & i & 1+i \\ a & 0 & b \\ 1-i & 2+i & 1 \end{bmatrix}$$

is Hermitian. (2)

b) Are there values of $a \in \mathbf{C}$ for which the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & a \end{bmatrix}$$

is unitary ? Justify your answer. (3)

c) Let $(x_1, x_2, x_3)$ and $(y_1, y_2, y_3)$ represent the coordinates with respect to the bases $B_1 = \{(1,0,0), (0,1,0), (0,0,1)\}$, $B_2 = \{(1,0,0), (0,1,2), (0,2,1)\}$. If $Q(X) = x_1^2 + 2x_1x_2 + 2x_2x_3 + x_2^2 + x_3^2$, find the representation of $Q$ in terms of $(y_1, y_2, y_3)$. (3)

7) a) Apply the Gram-Schmidt diagonalisation process to find an orthonormal basis for the subspace of $\mathbf{C}^4$ generated by the vectors

$$\{(1,i,0,1), (1,0,i,0), (-i,0,1,-1)\}$$ (6)

b) Find the orthogonal canonical reduction of the quadratic form $x^2 - 2y^2 + z^2 + 2xy + 6yz$ and its principal axes. Also, find the rank and signature of the quadratic form. (6)