MTE-13

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

DISCRETE MATHEMATICS

(Valid from 1st January, 2024 to 31st December, 2024)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(For January 2024 cycle)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

……

COURSE CODE **:** ……

COURSE TITLE **:** ……

ASSIGNMENT NO. **:** ……..…

STUDY CENTRE **:** ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto **December, 2024**. If you have failed in this assignment or fail to submit it by **December, 2024**, then you need to get the assignment for the 2015 cycle and submit it as per the instructions given in that assignment booklet.

8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

(To be done **after** reading the course material.)

Course Code: MTE-13
Assignment Code: MTE-13/TMA/2024
Maximum Marks: 100

1. State whether the following statements are **true** or **false**. Justify your answer with a short explanation or a counter-example. $(2 \times 10 = 20)$
   (i) The contra- positive of the statement "$x^2 + y^2 = 0 \Rightarrow x = 0\ and\ y = 0$" is "$x \neq 0\ and\ y \neq 0 \Rightarrow x^2 + y^2 \neq 0$".
   (ii) $a_n = \frac{1}{9}[2^{n+1} + (-1)^n]^2$ is the solution of the recurrence relation
   $$\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}\,, (n \geq 1), a_0 = a_1 = 1.$$
   (iii) The number of integers between 1 and 360 which are relatively prime to 360 are 96.[Note that 1 is relatively prime to every positive integer.]
   (iv) The coefficient of $x^6y^5z^9$ in $(x + y + z)^{20}$ is $C(20,6).C(20,5).C(20,9)$.
   (v) If a graph has 6 vertices and 10 edges, then it can't be regular.
   (vi) Peterson graph is bipartite.
   (vii) Every edge of a Tree is a bridge.
   (viii) The independence number of C_5, i.e., $\alpha(C_5)$ is 2.
   (ix) The DNF of the expression $p(x,y,z) = (x' \wedge z)'$ is $(x \vee y \vee z') \wedge (x \vee y' \vee z')$.
   (x) The number 8 has atmost one self-conjugate partition.

2. (a) Find the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 0,\ x_i \geq -4, \forall\ i$ using the generating function technique. (2)
   (b) Let a_n be the number of n-digit binary sequences such that no two consecutive digits are zero. Find a recurrence relation for $a_n, n \geq 1$. (3)
   (c) Solve the recurrence relation $a_n = 2a_{n-1} + n(n-1), n \geq 1, a_0 = 1$, using the generating function technique. (3)
   (d) Look at the following pattern:

$$\begin{aligned} 1 &= 1^3 \\ 3+5 &= 2^3 \\ 7+9+11 &= 3^3 \\ &\vdots \\ \boxed{\qquad\qquad} &= 100^3 \end{aligned}$$

   (i) How many $'+'$ signs should be there in the box?
   (ii) What are the smallest and largest integers in the box? (4)
   (e) Construct the logic circuit of the Boolean expression $(x_1' \vee x_2' \wedge x_3) \vee (x_2 \wedge x_3')$, where x_1, x_2, x_3 are the inputs in that circuitry. (3)

3. (a) Solve the recurrence relation $a_{n+1} = a_n + n.2^n, (n \geq 0), a_0 = 1$, using the method of telescopic sums. (3)
   (b) Prove by contradiction that the inverse of a square matrix, if exists, is unique. (2)
   (c) Give a direct proof of the following statement:
   "Every skew-symmetric matrix of odd order is singular". (2)
   (d) For any propositions p, q and r, show that $\sim p \vee \sim q \rightarrow r \equiv p \wedge \sim (q \rightarrow r)$. (3)

4. (a) Let p, q and r be the statements as defined below:
   p : The Course is enjoyable.
   q : The presentation is stimulating.
   r : The material is significant.

Write each of the following in symbolic form.

(i) The material is significant and the presentation is stimulating, but the course is not enjoyable.

(ii) It is not the case that both the course is enjoyable and, at the same time, the presentation is not stimulating. (2)

(b) Find out which of the following are propositions and which are not. Give reasons for your answers. (3)

(i) There are a total of 8 questions in this assignment.

(ii) Wow, it's a beautiful evening!

(iii) I am not sure whether I can run as fast as you can.

(iv) The set $\{2, -1, 2, -1, 2, -1, \ldots\}$ is unbounded.

(v) The set $\mathbb{N}$ of natural numbers is equivalent to the set $\mathbb{Z}$ of integers.

(vi) Rajasthan is the largest state in area among the Indian states and union territories.

(c) Using the principle of mathematical induction show that the sum of the cubes of three successive positive integers is divisible by 9. (2)

(d) Obtain the CNF of the Boolean expression $X(x_1, x_2, x_3) = x_2 \vee (x_1 \wedge x_3')$. (3)

5. (a) A Cantabil showroom offers 7 styles of pants. For each style, there are 10 different possible waist sizes, 6 pants lengths and 4 color choices. How many different types of pants could the showroom have? (2)

(b) Find the number of nonnegative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 12$.
And how many solutions in positive integers will it have? Justify. (3)

(c) What is the probability that an integer between 1 and 9,999 has exactly one 8 and one 9 ?Justify your answer. (3)

(d) Check wether the graph in Fig. 1 is Eulerian or not. Is it regular? Justify your answer. (2)

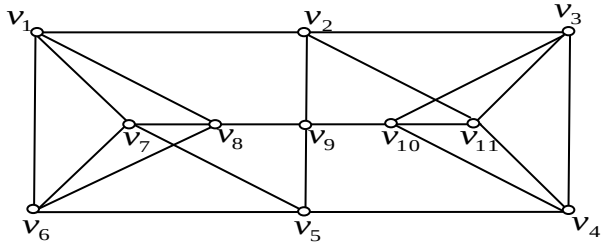
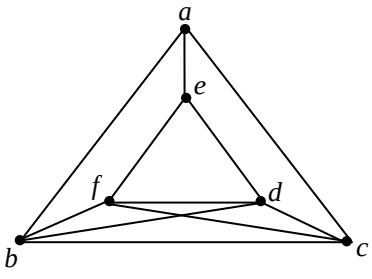
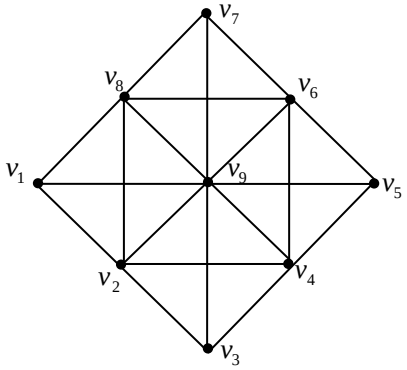


Fig. 1

(e) Check whether the graphs in Fig. 2 are planar or not. If yes, give their plane drawing, otherwise give a $K_{3,3}$ or K_5 subdivision. (5)



(*i*)



(*ii*)

Fig. 2

6. (a) Using the method of generating functions, solve the recurrence relation $a_n = 2a_{n-1} + 2^n,\ n \geq 1, a_0 = 1$. (3)

(b) Using combinatorial arguments, prove that

$$(a+b+c)^n = \sum_{r+s+t=n} \binom{n}{r,\ s,\ t} a^r b^s c^t.$$

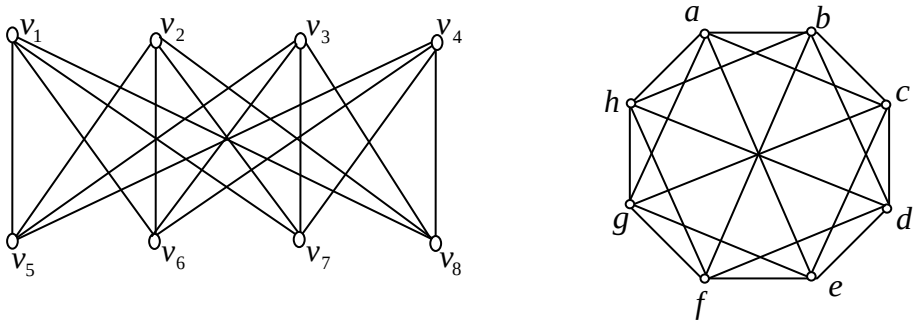
What is the coefficient of $x^{50}y^{12}z^{38}$ in $(x-y+2z)^{100}$? (4)

(c) Let G be an acyclic graph with n vertices and $n-1$ edeges. Show that G is connected. (3)

(d) How will you define a complete bipartite graph? Give an example of it. Is every complete bipartite graph complete? Justify. (2)

(e) Using combinatorial arguments, prove that $\binom{n}{k}\binom{k}{r} = \binom{n}{r}\binom{n-r}{k-r}$, where the notation $\binom{n}{k}$ is used for the number of ways to choose a subset of k objects from a subset containing n objects. (3)

7. (a) Write down the degree sequences of the graphs in Fig. 3:

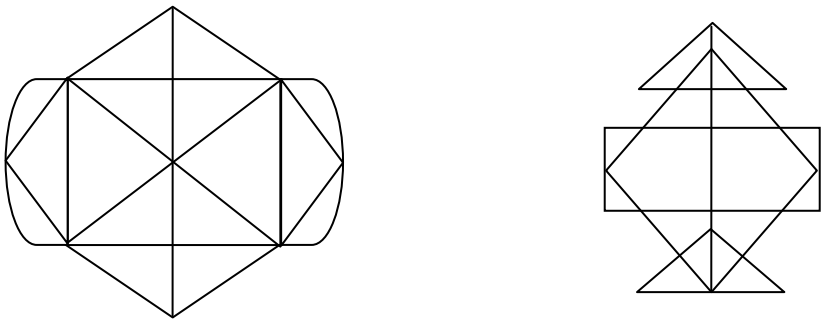


(i) (ii)

Fig. 3

Are these graphs isomorphic? Justify your answer. (4)

(b) Can the following figures be drawn without lifting the pen from paper and without covering any line segment more than once? Give reasons in support of your answers. (2)



(i) (ii)

Fig. 4

(c) How many colors are needed to color the 15 balls in the below given triangular array so that no two touching balls get same color? (3)

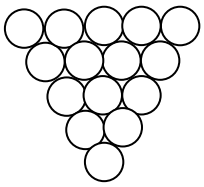


Fig. 5

(d) Prove that the graph given below is non-Hamiltonian. (3)

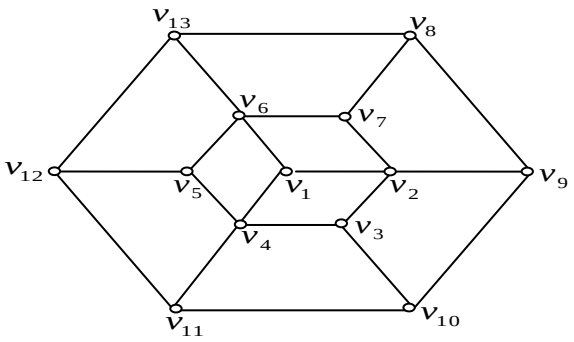


Fig. 6

(e) What is the edge-chromatic number of the graph given in Fig. 3(ii)? Justify by giving an explicit coloring. (3)