MTE-12

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

LINEAR PROGRAMMING

(Valid from 1st January, 2024 to 31st December, 2024)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: ………………………..….. **DATE:** ………………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: MTE-12
Assignment Code: MTE-12/TMA/2024
Maximum Marks: 100

1. Which of the following statements are true and which are false? Give reasons for your answer. (10)

   a) In an LP model, the feasible solution space can be effected when redundant constraints are deleted.

   b) If the primal LPP has an optimal solution, then the set of feasible solution to its dual is bounded.

   c) In a simplex iteration, an artificial variable can be dropped all together from the simplex table once the variable becomes non basic.

   d) The addition of a constant to all the elements of a payoff matrix in a two – person zero sum game can affect only the value of the game, not the optimal mix of the strategies.

   e) There may be a balanced transportation problem without any feasible solution.

2. a) A toy company manufactures two types of doll; a basic version-doll A and a deluxe version-doll B. Each doll of type B takes twice as long as to produce as one of type A, and the company would have time to make a maximum 2000 per day if it produce only the basic version. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. The company makes profit of ₹3 and 5 per doll respectively on doll A and B. How many of each should be produced per day in order to maximize profit? Solve this problem by graphical method. (6)

   b) Find all the basic solutions of the following system: (4)

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

3. a) A marketing manager has 5 salespersons and 5 sales districts. Considering the capabilities of the salespersons and the nature of the districts, the marketing manager estimates the sales per month (in thousand ₹) for each salesperson in each distinct as follows: (4)

		Districts				
		1	2	3	4	5
Salespersons	A	32	38	40	28	40
	B	40	44	28	21	36
	C	41	27	33	30	37
	D	22	38	41	36	36
	E	29	33	40	35	39

   Find the assignment of sales persons to districts that will result in maximum sales.

b) Find the maximum and minimax values of the following matrix game. (3)

$$\begin{array}{c} \\ A \end{array}\begin{array}{c} B \\ \begin{bmatrix} 1 & 0 & -1 & -2 \\ 2 & 3 & 4 & 0 \\ 1 & 2 & 5 & -3 \\ 3 & 4 & 2 & 1 \end{bmatrix} \end{array}$$

Does the matrix have a saddle point. Justify your answer.

c) The following table is obtained in the intermediate stage while solving an LPP by the simplex method. (3)

		-1	-2	0	0	0
1	x_1	1	2	-1	0	1
0	x_2	0	3	-1	1	2
		0	4	-1	0	1

Discuss whether an optimal solution will exist or not.

4. a) Solve by simplex method the following linear programming problem: (5)

Max $z = 2x + y + 2z$

s.t.

$3x - y + 2z \le 12$

$-2x + 4y \le 9$

$-x + 3y + 8z \le 15$

$x, y, z \ge 0.$

b) Show that the set of vectors (5)

$$a_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, a_3 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}.$$

from a basis for E^3.

5. a) Use the principle of dominance to reduce the size of the following game. Hence solve the game. (4)

$$\begin{bmatrix} 3 & 0 & 4 \\ 1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}.$$

b) i) Formulate the dual of the following problem: (6)

Minimize $z = 9x_1 + 12x_2 + 15x_3$

s.t. $2x_1 + 2x_2 + x_3 \ge 10$

$2x_1 + 3x_2 + x_3 \ge 12$

$$x_1 + x_2 + 5x_3 \geq 14$$
$$x_1, x_2, x_3 \geq 0$$

ii) Check whether $(2,2,2)$ is a feasible solution to the primal and $\left(\frac{1}{3}, 3, \frac{7}{3}\right)$ is a feasible solution to the dual.

iii) Use duality to check whether $(2,2,2)$ is an optimal solution to the primal.

6. a) Solve, graphically, the game whose pay-off matrix is: (6)

$$\begin{array}{c|cccc|} & & P_2 & & \\ \hline P_1 & 1 & 3 & -3 & 7 \\ & 2 & 5 & 4 & -6 \\ \hline \end{array}$$

b) Find an initial basic feasible solution for the following transportation problem using matrix-minima method. Also find the transportation cost. (4)

1	2	3	10
4	5	6	5
7	8	9	8
6	7	4	

7. a) Using the initial basic feasible solution for the transportation problem given below, find and optimal solution for the problem. (6)

	1	2	3	4	Availability
I	1 (40)	2	3	3 (30)	70
II	2	4 (28)	1	1 (10)	38
III	1	2	3 (30)	2 (2)	32
Requirements	40	28	30	42	

b) Test the following set for convexity. (4)

$S = \{(x, y) : x + y \leq 8 \text{ or } 2x + y \leq 10, x \geq 0, y \geq 0\}$.

8. a) Using the principle of dominance, solve the game whose pay-off matrix is given below: (6)

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	4	3	5	1
	A_2	4	5	3	5
	A_3	5	3	5	1
	A_4	1	5	1	9

b) Without sketching the region, check whether $P(0,0)$ is in the convex hull of the points $A(-1,-1), B(1,0)$ and $C(0,1)$. If it is in the region, write P as convex combination of A, B and C. (4)

9. a) A businessman has to get 5 cabinets, 12 desks and 18 shelves cleaned. He has two part-time employees, Anjali and Arnav. Anjali can clean 1 cabinet, 3 desks and 3 shelves in a day, while Arnav can clean 1 cabinet, 2 desks and 3 shelves in a day. Arnav is paid ₹22 per day and Anjali is paid ₹ 25 per day. Formulate the problem of finding the number of days for which Anjali and Arnav have to be employed to get the cleaning done with minimum cost as a linear programming problem. (5)

b) For the following pay-off matrix, transform the zero-sum game into an equivalent linear programming problem: (5)

		Player B		
		B_1	B_2	B_3
Player A	A_1	1	−1	3
	A_2	3	5	−3
	A_3	6	2	−2

10. a) Solve the following LP problem by using two-phase simplex method: (5)

Minimize $z = x_1 - 2x_2 - 3x_3$

subject to $-2x_1 + 3x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0$

b) The initial basic feasible solution of a transportation problem is given below: (5)

	W_1	W_2	W_3	Supply
F_1	16 (180)	20	12 (20)	200
F_2	14	8 (120)	18 (40)	160
F_3	26	24	16 (90)	90
Demand	180	120	150	450

Check whether the given solution is optimal. If it is not, then find the optimal solution.