MTE-11

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

PROBABILITY AND STATISTICS

(Valid from 1st January, 2024 to 31st December, 2024)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

……

COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: MTE-11
Assignment Code: MTE-11/TMA/2024
Maximum Marks: 100

1. Which of the following statements are ***True*** or ***False***? Give reasons for your answer. (10)

   i) If $y = a\,x - b,$ then the correlation coefficient between x and y does not exist and if it exists it is equal to zero.

   ii) For a normal distribution mean, median and standard deviation are all equal.

   iii) If $x \geq y$ then $y - x$ assume only non-positive values and hence $E(x) \leq E(y)$.

   iv) If two unbiased dice are rolled, then the probability of their same score being 6 is $\frac{1}{6}$.

   v) If the random variable x follows a normal distribution with known mean μ and unknown variance σ^2 then $\frac{x-\mu}{\sigma}$ is a statistic but $(x-\mu)$ is not.

2. a) The first three moments of a distribution about the value 2 are 1, 16 and – 40 respectively. Examine the Skewness of the distribution. (3)

   b) Find the maximum likelihood estimator for the parameter λ of the Poisson distribution on the basis of a sample of size n. Also, find its variance. (4)

   c) If x has an exponential distribution with parameter θ. Find the density function of $\log_e x$. (3)

3. a) If $x \sim n(\mu, \sigma^2)$. Find the moment generating function of $x - c$ where c is constant. (3)

   b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 3 times and H_0 is rejected if more than 2 heads are obtained. Find the probability of type I and type II errors. Also obtain the power of the test. (5)

   c) An unbiased die is rolled twice. Let A_1 denote the event: odd face roll on the first die, A_2 denote the event that total score is Odd. Check the independence of A_1 and A_2. (2)

4. a) Let $f(x, y) = x + y; 0 < x < 1, 0 < y < 1$. Find (i) the correlation coefficient between x and y, and (ii) $E(y/x)$. (6)

   b) Show that variance can be expressed in terms of the mutual differences $x_i - x_j$ of the observations i.e.

   $$S^2 = \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_j)^2.$$ (4)

5. a) The joint probability distribution of x and y is given below: (4)

y \ x	0	1	2
0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{24}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{40}$
2	$\frac{1}{8}$	$\frac{1}{20}$	—
3	$\frac{1}{120}$	—	$\frac{1}{120}$

Find (i) $P(x=1, y=2)$

(ii) $P(x=0, 1 \le y < 3)$

(iii) $P(x+y \le 1)$

(iv) $P(x > y)$

b) Every clinical thermometer is classified into one of the four catergories A, B, C and D on the basis of inspection and test. From the past experience it is known that thermometers produced by a certain manufacturer are distributed among the four categories in the following proportions: (6)

Category:	A	B	C	D
Proportion:	0.87	0.09	0.03	0.01

A new lot of 1336 thermometers is submitted by the manufacture for inspection and test and the following distribution of categories obtained:

Category:	A	B	C	D
No. of thermometers Reported:	1188	91	47	10

At 5% level of significance test whether this new lot of thermometers differ from the previous experience.

6. a) The following information about advertising expenditures and sales of a company is given below: (6)

	Advertising expenditure (x) Lakhs (₹)	Sales (y) Lakhs (₹)
Average	10	90
Variance	9	144
Correlation Coefficient between x and y is 0.8.		

What should be the advertising budget if the company wants to attain sales target of ₹ 120 Lakhs?

b) The police plans to enforce speed limits by using radar traps of 4 different locations within the city. The radar traps at each of the locations L_1, L_2, L_3 and L_4 are operated $40\%, 30\%, 20\%$ and 30% of time. If a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5 and 0.2 respectively of passing through these locations. What is the probability that the person will receive a speeding challan. (4)

7. a) Find the mean and standard deviation for the following data: (5)

Class interval	Frequency
0 – 10	5
10 – 20	10
20 – 30	24
30 – 40	15
40 – 50	6

b) State Chebychev's inequality. Hence obtain the lower bound for $P[-1 < x < 9]$ if the $E(x)$ and $E(x^2)$ of x are 4 and 20 respectively. (5)

8. a) Let X be a single observation from the p.d.f.

$$f(x, \theta) = \theta e^{-\theta x}, 0 \le x < \infty.$$

If $X \ge 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta = 1$, obtain the values of type I and type II errors. (5)

b) The probability that a student passes a Physics test is $\frac{2}{3}$ and the probability that the student passes both a Physics test and an English test is $\frac{14}{15}$. The probability that the student passes at least one test is $\frac{4}{5}$. What is the probability that the student passes the English test? (2)

c) Draw the cumulative frequency curves for the following distribution:

Marks	No. of Students
0 – 10	4
10 – 20	8
20 – 30	11
30 – 40	15
40 – 50	12
50 – 60	6
60 – 70	3

From the graph, obtain the median. (3)

9. a) X_1, X_2 and X_3 is a random sample of size 3 from a population with mean μ and variance σ^2. T_1, T_2 and T_3 are the estimators to estimate μ, and are given by

$T_1 = X_1 + X_2 - X_3$; $T_2 = 2X_1 + 3X_2 - 4X_2$ and $T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$. (5)

i) Are T_1 and T_2 unbiased? Give reason.

ii) Find the value of λ such that T_3 is unbiased.

iii) Which is the best estimator? State giving reasons.

b) A group of 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$ has been divided into two groups. The first has 100 items with mean 15 and standard deviation 3. Find the standard deviation of the second group. (5)

10. a) A single observation was taken from a population with p.d.f.

$$f(x, \theta) = \frac{2}{\theta^2}(\theta - x), \quad 0 \le x \le \theta.$$

Obtain $100(1-\alpha)\%$ confidence interval for θ. (5)

b) For 10 observations on price (X) and supply (Y) the following data were obtained (in appropriate units): (5)

$$\sum X = 130, \sum Y = 200, \sum X^2 = 2288, \sum Y^2 = 5506 \text{ and } \sum XY = 3467.$$

Obtain the line of regression of Y on X and estimate the supply when price is 16 units.