MTE-09

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

REAL ANALYSIS

Valid from 1st January, 2024 to 31st December, 2024

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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THE PEOPLE'S UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ……………………….….. **DATE:** ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-09
Assignment Code: MTE-09/TMA/2024
Maximum Marks: 100

1. Are the following statements **true** or **false?** Give reasons for your answer.

   a) Complement of the open interval $]0,1]$ is an open set.

   b) Every bounded sequences is not convergent.

   c) The function $f:[-2, 2] \to \mathbf{R}$ defined by $f(x) = \dfrac{4x+3}{x^2+1}$ is uniformly continuous.

   d) If the first derivative of a function at a point vanishes, then it has an extreme value at that point.

   e) The function $f:[0, 2] \to \mathbf{R}$ defined by $f(x) = x + [x]$ is not integrable. (10)

2. a) Determine the points of discontinuity of the function f and the nature of discontinuity at each of those points:

$$f(x) = \begin{cases} -x^2, & \text{when } x \le 0 \\ 4-5x, & \text{when } 0 < x \le 1 \\ 3x-4x^2, & \text{when } 1 < x \le 2 \\ -12x+2x, & \text{when } x > 2 \end{cases}$$

   Also check whether the function f is derivable at $x = 1$. (5)

   b) Find the following limit

$$\lim_{x\to 0} \frac{1-\cos x^2}{x^2 \sin x^2}$$ (3)

   c) Check whether the intervals $]5,9]$ and $[6,12[$ are equivalent or not. (2)

3. a) Prove that a strictly decreasing function is always one-one. (3)

   b) Write the inequality $4 \le 2x+3 \le 6$ in the modulus form. (2)

   c) Verify Bozano–Weierstrass Theorem for the following sets:

      i) Set of non-negative integers.

      ii) Interval $[-1, \infty]$ (3)

   d) Check whether the limit $\lim_{x\to 0}(x \operatorname{cosec} x)^x$ exists or not? (2)

4. a) Test the following series for convergence,

(i) $\sum_{n=1}^{\infty} n x^{n-1}, x > 0$.

(ii) $\sum_{n=1}^{\infty}\left[\sqrt{n^4+9}-\sqrt{n^4-9}\right]$ (6)

b) Show that $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5}{7n+2}$ is conditionally convergent. (4)

5. a) Determine the local minimum and local maximum values of the function f defined by $f(x)=3-5x^3+5x^4-x^5$. (5)

b) Show that $\mathbf{R_n}(x)$, the lagrange's form of remainder in the Maclaurin series expansion of e^{4x}, tends to zero as $n \to \infty$. Hence obtain the Maclaurin's infinite expansion for e^{4x}. (5)

6. a) If the partition P_2 is a refinement of the partition P_1 of $[a,b]$, then $L(P_1,f) \le L(P_2,f)$ and $U(P_2,f) \le U(P_1,f)$. Verify this result for the function $f(x)=2\cos x$ defined over the interval $\left[0,\frac{\pi}{2}\right]$ and the partitions $P_1=\left\{0,\frac{\pi}{3},\frac{\pi}{2}\right\}$ and $P_2=\left\{0,\frac{\pi}{6},\frac{\pi}{3},\frac{\pi}{2}\right\}$. (6)

b) Evaluate: $\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n^2}{(2n+r)^3}$. (4)

7. a) Use Cauchy's mean value theorem to prove that:

$$\frac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta}=\tan\theta, 0<\alpha<\theta<\beta<\frac{\pi}{2}$$ (5)

b) Find a and b such that $\lim_{x\to 0}\frac{a\tan x+bx}{x^3}$ exists. (3)

c) Show that $5+\sqrt{2}$ is an algebraic number. (2)

8. a) Using the principle of mathematical induction, show that

$$1^2+3^2+5x+...+(2n-1)^2=\frac{1}{3}n(4n^2-1)\ \forall n \in \mathbf{N}.$$ (4)

b) Show that the equation $x^3+x^2-2x-2=0$ has a real root other than $x=-1$. (4)

c) Check whether the set of integers is countable or not. (2)

9. a) Using Weiestrass M-test, show that the following series converges uniformly.

$$\sum_{n=1}^{\infty} n^3\ x^n, x \in\left[-\frac{1}{3},\frac{1}{3}\right].$$ (5)

b) Use the Fundamental Theorem of Integral Calculus to evaluate the integral

$$\int_0^1 \left(2x\sin\frac{1}{x} - \cos\frac{1}{x}\right)dx. \qquad (5)$$

10. a) Apply Bonnet Mean Value Theorem for integrals to show that

$$\left|\int_7^{10} \frac{\sin x}{x}\,dx\right| \le \frac{2}{7} \qquad (3)$$

b) Show that the function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 2x + 7$ has an inverse by applying the inverse function theorem. Find its inverse also. (3)

c) Verify the second mean value theorem for the function $f(x) = x$ and $g(x) = \cos x$ in the interval $\left[0, \frac{\pi}{2}\right]$. (4)