MTE-08

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

DIFFERENTIAL EQUATIONS

(Valid from 1st January, 2024 to 31st December, 2024)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:…………………………………………

NAME :…………………………………………

ADDRESS :…………………………………………

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COURSE CODE : …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: …………………………….…

STUDY CENTRE : ……………………….….. DATE :……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-08
Assignment Code: MTE-08/TMA/2024
Maximum Marks: 100

1. State whether the following statement are true or false. Justify your answer with the help of a short proof or a counter-example. (10)

   i) The initial value problem

   $$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 0$$

   has a unique solution in some interval of the form $-h < x < h$.

   ii) The orthogonal trajectories of all the parabolas with vertices at the origin and foci on the x-axis is $x^2 + 2y^2 = c^2$.

   iii) The normal form of the differential equation

   $$y'' - 4xy' + (4x^2 - 1)\, y = -3e^{x^2} \sin 2x \text{ is}$$

   $$\frac{d^2v}{dx^2} + v = -3 \sin 2x \text{, where } v = y\, e^{-x^2}.$$

   iv) The solution of the pde $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z^2$ is $z = -[y + f(x - y)]$.

   v) The pde $u_{xx} + x^2\, u_{xy} - \left(\frac{x^2}{2} + \frac{1}{4}\right) u_{yy} = 0$ is hyperbolic in the entire xy-plane.

2. a) Solve $\frac{dy}{dx} + xy = y^2\, e^{x^2/2} \sin x$. (3)

   b) Write the ordinary differential equation (3)

   $$y\, dx + (xy + x - 3y)\, dy = 0$$

   in the linear form, and hence find its solution.

   c) Given that $y_1(x) = x^{-1}$ is one solution of the differential equation (4)

   $$2x^2 y'' + 3xy' - y = 0, \quad x > 0,$$

   find a second linearly independent solution of the equation.

3. a) Solve, using the method of variation of parameters (5)

   $$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}.$$

   b) Solve the following equation by changing the independent variable (5)

   $$(1 + x^2)^2\, y'' + 2x(1 + x^2)\, y' + 4y = 0.$$

4. a) Find the integrating factor of the differential equation (3)

   $$(6xy - 3y^2 + 2y)\, dx + 2(x - y)\, dy = 0$$

   and hence solve it. .

b) Solve the equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x^m$, for all positive integer values of m. (3)

c) Solve the following IVP (4)

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = -6\sin 2x - 18\cos 2x$$

$$y(0) = 2,\ y'(0) = 2.$$

5. a) Solve: $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 5y = e^x \sec x$. (4)

b) Find the charge on the capacitor in an RLC circuit at $t = 0.01$ sec. when $L = 0.05$ Henry, $R = 2$ ohms, $C = 0.01$ Farad. $E(t) = 0$, $q(0) = 5$ Columbus and $i(0) = 0$. (3)

c) Solve: $x\frac{dy}{dx} + y \ln y = x y e^x$. (3)

6. a) Solve the following DEs

(i) $\left(\frac{dy}{dx} - 1\right)^2 \left(\frac{d^2y}{dx^2} + 1\right)^2 y = \sin^2(x/2) + e^x + x$. (3)

(ii) $2x^2y\left(\frac{d^2y}{dx^2}\right) + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right)$. (4)

b) The differential equation of a damped vibrating system under the action of an external periodic force is: (3)

$$\frac{d^2x}{dt^2} + 2m_0\frac{dx}{dt} + n^2x = a\cos pt$$

Show that, if $n > m_0 > 0$ the complementary function of the differential equation represents vibrations which are soon damped out. Find the particular integral in terms of periodic functions.

7. a) Verify that the Pfaffian differential equation (3)

$$yz\ dx + (x^2y - zx)\ dy + (x^2z - xy)\ dz = 0$$

is integrable and hence find its integral.

b) Solve the following equation by Jacobi's method (3)

$$x^2\frac{\partial u}{\partial x} - \left(\frac{\partial u}{\partial y}\right)^2 - a\left(\frac{\partial u}{\partial z}\right)^2 = 0.$$

c) Show that $2z = (ax + y)^2 + b$, where a, b are arbitrary constants is a complete integral of $px + qy - q^2 = 0$. (4)

8. a) Solve the following differential equations (6)

(i) $x^2p + y^2q = (x + y)\ z$.

(ii) $\sqrt{p} - \sqrt{q} + 3x = 0$.

b) Find the equation of the integral surface of the differential equation (4)

$$(x^2 - yz)\ p + (y^2 - zx)\ q = z^2 - xy$$

which passes through the line $x = 1,\ y = 0$.

9\. a) Using the method of separation of variables, solve $u_{xt} = e^{-t} \cos x$ when $u(x, 0) = 0,\ \frac{\partial u}{\partial t}(0, t) = 0$. (3)

b) Find the temperature in a bar of length ℓ with both ends insulated and with initial temperature in the rod being $\sin \frac{\pi x}{\ell}$. (7)

10\. a) Solve the following differential equations

(i) $[D^3 - DD'^2 - D^2 + DD']\ z = 0$. (2)

(ii) $[D^4 - D'^4 - 2D^2 D'^2]\ z = 0$. (2)

(iii) $[D^2 - 2DD' + D'^2]\ z = 12xy$. (3)

b) Show that the wave equation $a^2 u_{xx} = u_{tt}$ can be reduced to the form $u_{\xi\eta} = 0$ by the change of variable $\xi = x - at,\ \eta = x + at$. (3)