

**MTE-04**

**ASSIGNMENT BOOKLET**

**Bachelor's Degree Programme**

**ELEMENTARY ALGEBRA**

**(Valid from 1<sup>st</sup> January, 2024 to 31<sup>st</sup> December, 2024)**

**It is compulsory to submit the assignment before filling in the exam form.**



**School of Sciences  
Indira Gandhi National Open University  
Maidan Garhi  
New Delhi-110068  
(2024)**

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

### Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

- 1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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**ROLL NO.:** .....

**NAME:** .....

**ADDRESS:** .....

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**COURSE CODE:** .....

**COURSE TITLE:** .....

**ASSIGNMENT NO.:** .....

**STUDY CENTRE:** ..... **DATE:** .....

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**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) **This assignment is valid only upto December, 2024.** If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.
- 7) It is compulsory to submit the assignment before filling in the exam form.

**We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

## ASSIGNMENT

Course Code: MTE-04  
Assignment Code: MTE-04/TMA/2024  
Maximum Marks: 100

- 1) Which of the following statements are true? Justify your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that ‘{1, padma, blue} is a set’ is true, you need to say that this is true because it is a well-defined collection of 3 objects.) (20)
- i) Eliminating  $z$  from  $x + 2y + 3z = 2$ ,  $3x + 2y + 3z = 6$  and  $2x + 3y = 5$  gives  $x + 2y = 2$ .
- ii) The roots of  $x^3 - 8x - 3 = 0$  are given by  $x = \frac{8 \pm \sqrt{64 + 12}}{2}$ .
- iii)  $\left(\sqrt{2}, 1, \frac{3}{5}\right) \in \mathbb{Q} \times \mathbb{Z} \times \mathbb{R}$ .
- iv) Given any  $n$  positive numbers in  $\mathbb{R}$ , the product of their harmonic mean and their arithmetic mean is 1.
- v) If  $A$  and  $B$  are two sets such that  $(A \cup B)^c$  is empty, then either  $A = \emptyset$  or  $B = \emptyset$ .
- vi) For any  $x, y \in \mathbb{R}$ ,  $|x - y| \geq ||x| - |y||$ .
- vii) The geometrical representation of the set  $\{ix \mid x \in \mathbb{R}\}$  is a point.
- viii) Any finite set is a subset of  $\mathbb{Z}$ .
- ix) Every biquadratic equation has at least one real root.
- x) The converse of the statement, ‘Every student of MTE-04 has completed FST-01’, is ‘Every student of FST-01 has completed MTE-04’.
- 2) a) Show that  $1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > 2\sqrt{n+1} - 2 \forall n \in \mathbb{N}$ . (5)
- b) Let  $a, b > 0$ ,  $a + b = 1$ ,  $n > 1$ . Show that  $\left(a + \frac{1}{a}\right)^n + \left(b + \frac{1}{b}\right)^n \geq \frac{5^n}{2^{n-1}}$ . (5)
- 3) a) Using the discriminant, give the nature of the roots of  $7x^3 + x^2 - 35x = 5$ . Also solve the equation. (8)
- b) Find the cubic equation whose roots are the cubes of the roots of  $x^3 + ax^2 + bx + c = 0$ ,  $a, b, c \in \mathbb{R}$ . (4)
- c) Obtain the resolvent cubics, by Descartes’ method and by Ferrari’s method, of the equation  $x^4 + 4x^3 + 8 = 0$ . Are the cubics the same? Further, use either method to obtain the roots of this equation. (13)

- 4) a) If  $A$  and  $B$  are the set of even integers and set of odd integers, respectively, find  $A \cup B$  and  $(A \cup B)^c$ . (2)
- b) i) Find  $A \times B$ , and the number of elements in it, where  
 $A = \{3n + 2 \mid 1 \leq n \leq 10\} \subseteq \mathbb{Z}$ , and  
 $B = \{n \in \mathbb{Z} \mid 1 \leq n \leq 15\} \cap \{m \in \mathbb{Z} \mid 2 \mid m\}$ .
- ii) Given any two sets  $C$  and  $D$ , under what conditions on them will  $C \times D$  and  $D \times C$  have the same number of elements? Give reasons for your answer. (5)
- c) Express the following situation in a Venn diagram:  
 In a survey of 60 women, it is found that 25 have studied upto Class 12 only, 10 have studied till Class 10 only, 26 got scholarships, 9 of those studying till Class 12 got scholarships, 8 of those studying till Class 10 got scholarships, and 11 had completed their BA degree. (3)
5. In the context of your IGNOU studies, give the following: (5)
- i) an example of an implication;  
 ii) the converse of your statement in (i) above;  
 iii) the contrapositive of your statement in (i) above;  
 iv) a statement using  $\forall$ ;  
 v) a statement using  $\exists$ .
6. a) Give the following: (3)
- i) a  $2 \times 4$  matrix;  
 ii) the transpose of the matrix in (i) above;  
 iii) a system of linear equations represented by  $AX = B$ , where  $A$  is the matrix in (ii) above.
- b) Consider the linear system
- $$\begin{aligned} 2x - 3y + 4z &= 20\frac{2}{3} \\ x + 2y - 3z + 13.5 &= 0 \\ -x - 2y + 5z &= \frac{113}{6} \end{aligned}$$
- Give the **two reasons** for Cramer's Rule being applicable for solving this system. Also use the rule to solve the linear system. (7)
7. a) Find the values of  $a \in \mathbb{R}$  for which  $ai$  is a solution of  $z^4 - 2z^3 + 7z^2 - 4z + 10 = 0$ . Also find all the roots of this equation. (5)
- b) Find all the 8<sup>th</sup> roots of  $3i - 3$ . Also show any one of them in an Argand diagram. (5)
8. a) Using the method of substitution, obtain the solution set in  $\mathbb{R}^3$ , of the following:
- i)  $x - \pi = 5$

ii)  $2x - y + z = 1, x - 2y + z = 3, y = \sqrt{2} - z$   
iii)  $x - y = 5, x = 7, 2x - 3y = 5$  (7)

- b) Give a real life situation problem, which is mathematically translated into  
 $2x + y + 2z = 18, x + 3y + 3z = 24, 3y = 6.$

Also, explain how this linear system models your problem. (3)