MTE-03

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

MATHEMATICAL METHODS

(Valid from 1st January, 2024 to 31st December, 2024)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

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THE PEOPLE'S UNIVERSITY

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Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

1. State whether the following statements are ***true*** or ***false*** giving reasons in support of your answer. (10)

   i) A binomial distribution has mean 3 and variance 4.

   ii) The function $f(x) = x^3$ has no maxima or minima.

   iii) The line of regression of x on y is the same as the line of regression of y on x.

   iv) The plane $x + 2y - z = 5$ is parallel to the line $\dfrac{x}{1} = \dfrac{y-5}{-1} = \dfrac{z+1}{-1}$.

   v) A continuous random variable can have probability density function

   $$f(x) = \begin{cases} 4x^2, & 0 < x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

2. a) Define $f : \boldsymbol{R} \to \boldsymbol{R} : f(x) = \dfrac{x^2}{1+x^2}$. Is f injective, surjective, monotone? (3)

   b) A bag contains 3 white balls and 2 red balls. Another bag contains 5 white and 3 red balls. A bag is chosen at random and a ball is drawn from it. Find the probability that it is white. (3)

   c) Compute the correlation co-efficient between X and Y for the following data: (4)

X	9	7	6	1	3	9	4
Y	1	3	5	6	9	6	4

3. a) Find asymptotes of the graph of the function $y = \dfrac{x^2 - 8}{x - 1}$. (3)

   b) Find the term independent of a in the binomial expansion of $\left(3a - \dfrac{4}{a^6} \right)^7$. (3)

   c) In a certain Poisson distribution the probability of 3 successes is exactly equal to the probability of 4 successes. Find its mean and standard deviation. Also find the probability of more than 1 success for the given distribution. (4)

4. a) Integrate $\displaystyle\int_0^1 \tan^{-1}\left(\dfrac{2x}{i - x^2} \right) dx.$. (3)

b) Find the mean and the standard deviation of a random variable with the following probability density function:

$f(x) = e^{-x}, x \geq 0.$ (4)

c) A club has 9 member having ages 21, 28, 23, 29, 52, 43, 32, 37 and 30 years. One has to be at least 30 years of age to be eligible for the presidentship of the club. A simple random sample of size 5 is selected to provide an estimate of the population proportion eligible for presidentship. Find the mean and the standard error of this estimate. (3)

5. a) Find the radius of the sphere which passes through the points $(0,0,1), (1,0,0,), (0,1,0)$ and $(0,0,1)$. (3)

b) Verify Euler's theorem for the function $f(x, y) = \dfrac{x+y}{x-y}$. (3)

c) Two groups of 10 plants each were grown on two different fertilizers. The average height of first group of plants was 92.44 cm, with a standard deviation of 4 cm. For the second group, the average height was 90 cm with a standard deviation of 2 cm. At 5% level of significance test the hypothesis that first fertilizer is better than the second in terms of the plant growth.

[Given : $t_{0.05,18} = 1.734, t_{0.05,20} = 1.729, t_{0.05,22} = 1.721$] (4)

6. a) Find $(a \times b) \cdot c$ where

$a = 2i + 3j - k, b = i - 2j + k, c = 3i + 4j + 5k.$ (3)

b) Solve the differential equation $\cos x \dfrac{dy}{dx} + \sin xy = xe^{2x} \cos^2 x.$ (3)

c) A garden pea plant is genetically mixed for the gene pair Tt, where the gene T (for tall) is dominant over the gene t (for short). The plant produced 40 tall and 20 short offspring. Using χ^2-test find out whether the plant was self fertilized or fertilized by a short plant at 5% level of significance. (4)

7. a) It is known that 10 men out of every 100 men and 30 women out of every 1000 women are color blind. In a community, half the population is male. Using Baye's theorem find the probability that a color blind person chosen at random from among all color blind persons in the community is male. (3)

b) Show that $\sin x(1 + \cos x)$ has a maximum at $x = \dfrac{\pi}{3}$. (4)

c) Find the sum of the first n terms of the series $\log 2 + \log 6 + \log 18 + \log 54 + \dots.$ (3)

8. a) If $f : R \to R$ is defined by $f(x) = 4x + 1$ then show that f is a bijection. Find the formula that defines f^{-1}. (3)

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b) Obtain $\int \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)dx.$ (3)

c) The difference of mean and variance of a binomial distribution of 9 trails is 4. Find the probability p of success of the binomial distribution. Also find the probability of (i) exactly two successes and (ii) less than 2 successes. (4)

9. a) A and B are two events which are independent. The probability that both A and B occur is $\dfrac{1}{6}$ and the probability that neither of them occurs is $\dfrac{1}{3}$. Find the probability of the occurrences of A and B. (3)

b) If $y = e^x + e^{-x}$, prove that $\sqrt{y^2 - 4}.$ (3)

c) Fit a straight line for regression of Y on X from the following table. (4)

x	0	1	2	3	4	5	6
y	2	1	3	2	4	3	5

Find the value of y when $x = 10$.

10. a) Calculate: (i) Quartile deviation and (ii) Mean Deviation from mean for the following data: (4)

Marks	No. of Students
15 – 25	4
25 – 35	11
35 – 45	14
45 – 55	18
55 – 65	8
65 – 75	5

b) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum. (3)

c) Find the angle between the planes $6x - 4y + 2z = 1$ and $3x + 12y - 9z = 2$. Also specify the type of the angle obtained. (3)