PHE-14

ASSIGNMENT BOOKLET
Bachelor's Degree Programme (B.Sc.)

MATHEMATICAL METHODS IN PHYSICS-III

Valid from January 1, 2023 to December 31, 2023

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

Please Note

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 64 credits of electives in these 3 disciplines, at least 16 credits out of those 64 credits should be from lab courses.**
- **You cannot appear in the Term-End Examination of any course without registering for the course. Otherwise, your result will not be declared and the responsibility will be yours.**

School of Sciences
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2023

Dear Student,

We hope you are familiar with the system of evaluation to be followed for the Bachelor's Degree Programme. At this stage you may probably like to re-read the section on assignments in the Programme Guide for Elective Courses that the University sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation which would consist of one tutor-marked assignment for this course.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your TMA answer sheet, please write the details exactly in the following format:

ENROLMENT NO. : ……

NAME : ……

ADDRESS : ……

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COURSE CODE : ……..

COURSE TITLE : ……..……

ASSIGNMENT NO. : ……..…

STUDY CENTRE : ……..……. DATE : ….……...……

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise and in your own words. Do not copy answers from study material.

5) While solving problems, clearly indicate the question number along with the part being solved. Write units at each step of your calculations as done in the text because marks will be deducted for such mistakes. Take care of significant digits in your work. Recheck your work before submitting it.

6) **This assignment will remain valid from January 1, 2023 to December 31, 2023**. However, you are advised to submit it within **12 weeks** of receiving this booklet to accomplish its purpose as a teaching-tool.

We strongly feel that you should retain a copy of your assignment response to avoid any unforeseen situation and append, if possible, a photocopy of this booklet with your response.

We wish you good luck.

Tutor Marked Assignment
MATHEMATICAL METHODS IN PHYSICS-III

Course Code: PHE-14
Assignment Code: PHE-14/TMA/2023
Max. Marks: 100

Note: Attempt all questions. Symbols have their usual meanings. The marks for each question are indicated against it.

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1. a) Determine the values of a, b, c when (5)

$$M = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$ is orthogonal.

b) Verify the Cayley-Hamilton theorem of the matrix. (10)

$$P = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

and hence obtain P^{-1}.

c) If A^{ij} is an antisymmetric tensor and B_i is a vector, show that $A^{ij} B_i B_j = 0$. (5)

d) What are the four conditions to be satisfied by the elements of a group? Show that the set of all complex numbers of unit magnitude $u(1) = \{z : |z| = 1\}$ forms a group. (5)

2. a) Using the method of residues, evaluate the contour integral (10)

$$\int_C \frac{e^{iz}}{(z^2+1)} dz$$

where C is defined by $|z| < 4$.

b) Using the method of residues, evaluate the integral (10)

$$\int_0^\infty \frac{dx}{x^4+1}$$

c) i) Show that the function $f(z) = z^3$ is analytic in the entire z-plane. (2)

ii) Obtain the Taylor series expansion of $\cos^2 z$ about z = 0. (3)

3. a) Obtain the Fourier cosine transforms of the function:

$$f(x) = \begin{cases} \cos x & -\pi/2 < x < \pi/2 \\ 0 & |x| > \pi/2 \end{cases}$$ (5)

b) Calculate the inverse Laplace transform of the function:

$$F(s) = \frac{3s+10}{s^2-25} \ .$$ (5)

c) Solve the initial value problem using the method of Laplace transforms:

$$y'' - 2y' - 3y = 0;\ y(0) = 1,\quad y'(0) = 7$$ (10)

d) Calculate the Laplace transform of $t^n \mathrm{e}^{at}$. (5)

4. a) Show that

i) $$x^{-n}\left[\frac{d}{dx}\left\{x^n J_n(x)\right\}\right] = J_{n-1}(x)$$ (5)

ii) Using the expression

$$J_m(x) = \sum_{K=0}^{\infty} (-1)^K \frac{1}{K!\Gamma(m+K+1)}\left(\frac{x}{2}\right)^{2K+m}$$

for Bessel function of the first kind of order *m*, show that

$$[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2 = \frac{2}{\pi x}$$ (5)

b) Using the generating function

$$g(x, t) = 1/\sqrt{1-2xt+t^2}$$

for Legendre polynomials show that:

$$nP_n(x) - xP_n'(x) + P_{n-1}'(x) = 0$$ (10)

c) Use Rodrigues' formula for Laguerre polynomials to generate $L_4(x)$. (5)

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