MTE-13

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

DISCRETE MATHEMATICS

(Valid from 1st January, 2023 to 31st December, 2023)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

……

COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2023. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: MTE-13
Assignment Code: MTE-13/TMA/2023
Maximum Marks: 100

1. Check whether the following statements are true or not. Justify your answers with a short proof or a counter example. (20)

   i) If the contrapositive of a statement is true, then the statement itself is also true.

   ii) $a_n + 3a_{n-1} + 2a_{n-2} = 2^n$ is a linear homogeneous recurrence relation.

   iii) A particular solution of the recurrence relation $a_n - 2a_{n-1} + a_{n-2} = 1$ has the form $C n^2$.

   iv) The edge chromatic number of the graph K_6 is 5.

   v) If a dice is rolled thrice, then the probability of getting a 6 each time is $\frac{1}{72}$.

   vi) Every odd cycle has the same chromatic and edge chromatic numbers.

   vii) Every Eulerian graph is Hamiltonian.

   viii) S_4^3 gives the number of ways in which any 3 objects can be placed in any 4 boxes.

   ix) There exists a self-complementary planar graph on 5 or more vertices.

   x) The number of partitions of 6 is 10.

2. a) Draw the logic circuit for the Boolean expression $((x_1 \wedge x_2)' \vee x_3)' \wedge x_2$. (2)

   b) Express the following statements in symbolic form. (3)

      i) There is a man in the park with blue eyes.

      ii) Every blue-eyed man in the park is wearing a red hat.

      iii) If a man wears no hat, then he has black eyes.

   c) Using generating functions find $S_n = 1 + 2 + 3 + ... + n$. (5)

3. a) There are about 77 crore ways to arrange the letters of the word "COMBINATORICS". Count the exact number of such ways. (2)

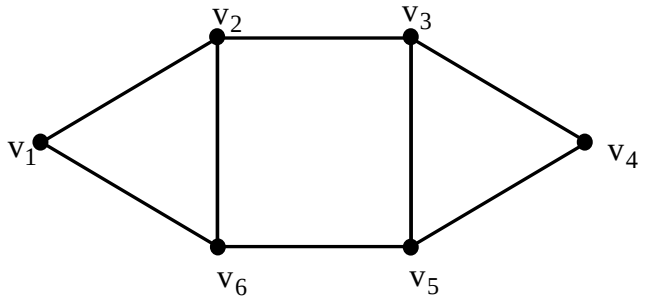
   b) Solve the recurrence relation:

   $$a_n - 6a_{n-1} + 9a_{n-2} = 3^n$$ (4)

   c) List all the onto mappings from the set $\{a,b,c,d\}$ to $\{1,2,3,4\}$. How many onto mappings are there form $\{a,b,c,d\}$ to $\{1,2,3,4,5\}$? (4)

d) For any statements p, q and r, prove that $(p \to q) \wedge (\sim q \to r) \wedge r \wedge \sim q \Rightarrow \sim p$. (5)

4. a) Draw three nonisomorphic induced subgraphs of the following graph, each having the same number of vertices. Justify your choice. (4)



b) Is the complement of the Peterson graph planar? Justify your answer. (3)

c) What do you understand by a subdivision of a graph? Is every subdivision of a Hamiltonian graph Hamiltonian? Justify. (3)

5. a) Let C_n denote the number of n-tuples whose entries are 0 or 1 only, and two consecutive entries of which are zero.
   i) Find C_1 and C_2.
   ii) Find a recurrence relation for C_n. (5)

b) write down and count all the partitions of the number 7. To verify your answer use the generating function for P_n, taking $n = 7$ in Theorem 5 (of Unit 5, Block2). (5)

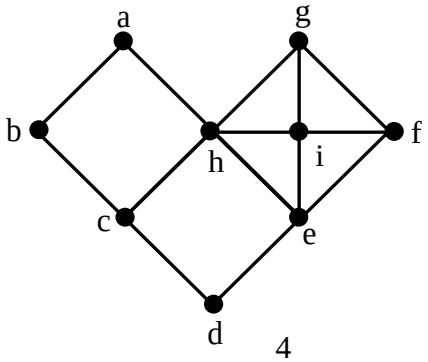
6. a) Express x^5 in terms of falling factorials and hence evaluate S_5^m for $m = 0,1,2,3,4,5$. (5)

b) Find a recurrence relation for a_n, the number of ways to arrange cars in a row with n spaces if we can use Maruti 800, Tata Safari or Scorpio. A Tata Safari or Scorpio requires two spaces, whereas a Maruti 800 requires just one space. Assume that you have unlimited number of each type of car and we do not distinguish between 2 cars of the same type. (4)

c) Define the nth Bell number. Using the formula for Bell numbers or otherwise, determine B_5. (3)

d) Show that if 7 colours are used to paint 50 bicycles and each bicycle is coloured with a single colour, at least 8 bicycles will have the same colour. (3)

7. a) Let T be a graph such that between every two vertices of it there is exactly one path. Show that T is a tree. (3)
   b) Define vertex connectivity and cut vertex set of any graph G. Find the vertex connectivity and cut vertex set for the following graph: (4)



c) How many numbers from 0 to 759 are not divisible by either 3 or 7? (3)

8. a) Solve the recurrence relation:

$a_n = 2a_{n-1} + 1$ if $n \geq 1$ and $a_0 = 0$,
using generating function technique. Also find a_5 using your answer. (5)

b) Is there a 4-regular graph on 7 vertices? Justify your answer. (2)

c) Find the Boolean expression in the DNF form for the function defined in tabular form below: (3)

x	y	z	$f(x, y, z)$
1	0	1	1
0	1	0	0
0	0	1	1
1	1	1	1
1	0	0	0
0	1	1	1
1	1	0	1
0	0	0	0