MTE-12

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

LINEAR PROGRAMMING

(Valid from 1st January, 2023 to 31st December, 2023)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
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Maidan Garhi
New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2023**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying all the blocks)

Course Code: MTE-08
Assignment Code: MTE-12/TMA/2023
Maximum Marks: 100

1. Which of the following statements are true and which are false? Give a short proof or a counter-example in support of your answer. (10)

   a) In a two-dimensional LPP solution, the objective function can assume the same value at two distinct extreme points.

   b) Both the primal and dual of an LPP can be infeasible.

   c) An unrestricted primal variable converts into an equality dual constraint.

   d) In a two-person zero-sum game, if the optimal solution requires one player to use a pure strategy, the other player must do the same.

   e) If 10 is added to each entry of a row in the cost matrix of an assignment problem, then the total cost of an optimal assignment for the changed cost matrix will also increase by 10.

2. a) Solve the following linear programming problem using simple method: (6)

$$\text{Maximize } z = 3x_1 + 5x_2 + 4x_3$$
$$\text{Subject to } \quad 2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
$$x_1,\ x_2,\ x_3 \ge 0$$

   b) Using the principle of dominance, reduce the size of the following game:

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

   Hence solve the game. (4)

3. a) Find all basic feasible solutions for the following set of equations: (6)

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$
$$x_1,\ x_2,\ x_3,\ x4 \ge 0.$$

   b) Examine convexity of the following sets: (4)

   i) $S_1 = \{(x_1,\ x_2) \in \mathbb{R}^2 \mid 4x_1 + 3x_2 \le 6,\ x_1 + x_2 \ge 1\}$

   ii) $S_2 = \{(x,\ y) \in \mathbb{R}^2 \mid x^2 + y^2 \ge 1\}$.

4. a) Solve the following linear programming problem by graphical method: (5)

$$\text{Maximize } \quad z = 5x_1 + 7x_2$$

Subject to $x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

$x_1, x_2 \ge 0.$

b) Find the dual of the following LPP: (5)

Maximize $z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \le 3$

$2x_2 - x_3 \ge 4$

$x_1, x_2 \ge 0$ and x_3 is unrestricted in sign.

5. a) Find the initial basic feasible solution of the following transportation problem using matrix-minima method:

	Destinations			Supply
	I	II	III	
Sources A	2	7	4	5
Sources B	3	3	1	8
Sources C	5	4	7	7
Sources D	1	6	2	14
Demand	7	9	18	34

Also, find the optimal solution. (5)

b) Solve the following game graphically: (5)

		Player 'B'	
		B_1	B_2
Player 'A'	I	2	7
	II	3	5
	III	11	2

6. a) A firm manufactures two types of products, A and B, and sells them at a profit of ₹ 2 on type A and ₹ 3 on type B. Each product is processed on two machines M_1 and M_2. Type a requires one minute of processing time on M_1 and two minutes on M_2; type B requires one minute on M_1 and one minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for 10 hours during any working day.
Formulate the problem at LPP. (4)

b) Solve the following assignment problem: (6)

	A	B	C	D	E
I	2	9	2	7	1
II	6	8	7	6	1
III	4	6	5	3	1
IV	4	2	7	3	1
V	5	3	9	5	1

7. a) The following table is obtained in the intermediate state while solving an LPP by simplex method:

	C_i's	30	23	29	0	0	
B	C_B	x_1	x_2	x_3	S_1	S_2	R.H.S.
S_1	0	0	2	$-9/2$	1	$-3/2$	31/2
X_1	30	1	1/2	5/4	0	1/4	7/4

Check whether an optimal solution of the LPP will exist or not. (3)

b) Write the LPP model of the following transportation problem: (3)

5	7	6	4	70
2	8	3	1	50
1	7	4	5	90
50	40	50	70	

c) Find the range of values of p and q which will render the entry (2, 2), a saddle point for the following game: (4)

	Player B		
Player A	2	4	5
	10	7	q
	4	p	6

8. a) Test the convexity of the following sets: (4)

$$S_1 = \{(x, y) \mid x^2 + y^2 \geq 1,\ y \geq x,\ y \geq -x\}$$

$$S_2 = \{(x, y) \mid x^2 + y^2 \leq 16,\ x \leq 2,\ y \geq 2\}.$$

b) Determine all the basic feasible solutions to the equations

$$x_1 + x_2 + 2x_3 = 4$$

$$2x_1 - x_2 + x_3 = 2.$$

Identify the degenerate basic feasible solutions. (6)

9. a) Let $A = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 5 & 1 \end{bmatrix}$. Compute AB, BC, AC, if they exists, otherwise, give reason for their non-existence. (2)

b) Solve the following LPP: (5)

Minimize $z = x_1 - 2x_2 - 3x_3$

Subject to $-2x_1 + 3x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0.$

c) Find the saddle point (if exists) in the following pay-off matrix:

	Player B		
Player A	-1	-1	7
	3	1	3
	6	-1	-3

Also, find the value of the game. (3)

10. a) Determine an initial basic feasible solution to the following transportation problem and hence find an optimal solution to the problem: (7)

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	12	10	5	7	8
O_3	9	11	8	13	10
	4	6	8	6	

b) Find all values of k for which the vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\end{bmatrix}$ and $\begin{bmatrix}k\\-k\\2\end{bmatrix}$ are linearly independent. (3)