MTE-04

# ASSIGNMENT BOOKLET

**Bachelor's Degree Programme**
**(B.Sc./B.A./B.Com.)**

# ELEMENTARY ALGEBRA

**(Valid from 1st January, 2023 to 31st December, 2023)**

**It is compulsory to submit the assignment before filling in the exam form.**

**School of Sciences**
**Indira Gandhi National Open University**
**Maidan Garhi**
**New Delhi-110068**
**(2023)**

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

**Instructions for Formatting Your Assignments**

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

---

**ROLL NO.:** ……………………………………………

**NAME:** ……………………………………………

**ADDRESS:** ……………………………………………

……………………………………………

……………………………………………

**COURSE CODE:** …………………………….

**COURSE TITLE:** …………………………….

**ASSIGNMENT NO.:** …………………………..…

**STUDY CENTRE:** …………………………..….. **DATE:** ……………………………….……………...

---

**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2023**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

**We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

## ASSIGNMENT

### (To be done after studying Blocks 1 and 2.)

**Course Code: MTE-04**
**Assignment Code: MTE-04/TMA/2023**
**Maximum Marks: 100**

1) Which of the following statements are true? Justify your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that '{1, padma, blue} is a set' is true, you need to say that this is true because it is a well-defined collection of 3 objects.)

   i) The collection of Venn diagrams is a set.

   ii) $(A \setminus B) \cup C = A \setminus (B \cap C)$ for any three sets $A$, $B$ and $C$.

   iii) If $z \in \mathbb{R}$, then $|z| = z$.

   iv) $x^m + a_1 x^{m-1} + \cdots + a_{m-1}x + a_m = 0$, $a_i \in \mathbb{R} \, \forall \, i = 1, \ldots, m$, has a root in $\mathbb{R}$ only if $m$ is an odd number.

   v) $x > 0$ is necessary for $x + 2 > 1$.

   vi) Any system of three linear equations in two variables has no solution.

   vii) If a matrix has $n^2$ entries, where $n \in \mathbb{N}$, then it is a square matrix.

   viii) For $n > 2$, the $AM$ of the first $n$ natural numbers is greater than $n+1$. (16)

2) a) For any two sets $A$ and $B$, in a universal set $U$, prove that $A \subseteq B \Leftrightarrow A \cup B = B$. (3)

   b) Draw a Venn diagram of sets $A$, $B$ and $C$ where $A \subseteq B$, $A \cap C \neq \phi$, $B \cap C = \phi$. What is the universal set you have chosen? Justify your choice of sets in the diagram. (4)

   c) Find $x$ and $y$, given that $(5x + y, 3x - y) = (2, 2x) \in \mathbb{Q} \times \mathbb{Q}$. (3)

3) a) Express $z = \dfrac{1}{-5-i}$ in standard (algebraic) form. Further, give an Argand diagram in which $z$, $\bar{z}$ and $-z$ are plotted. (4)

   b) Obtain the polar and exponential representations of $z_1$, $z_2$ and $z_1 z_2$, where $z_1 = \dfrac{1}{2} - 2i$ and $z_2 = 3 + i$ (4)

   c) Apply De Moivre's theorem to write $(\sqrt{3} + i)^5$ in the form $a + ib$, with $a, b \in \mathbb{R}$. (3)

   d) Find the sum of the $5^{th}$ roots of unity. (4)

4. a) Find the polynomial over $\mathbb{R}$ of least degree which has $i - 3$ and $\sqrt{7} + 5i$ as its roots. (2)

b) Obtain the discriminant of the equation $2x^3 - 23x^2 + 82x - 78 = 0$. Hence provide the nature of its roots. (5)

c) Find the roots of the equation $2x^3 - x^2 - 22x - 24 = 0$, if two of them are in the ratio $3:4$. (5)

d) Solve $x^4 - 8x^3 + 21x^2 - 20x + 5 = 0$ given that the sum of two of its roots is $4$. (7)

5. a) The annual bonus given to the employees of a company is 5% of their taxable incomes, after the state and central taxes are deducted. The state tax is 10% of taxable income. The central tax is 20% of taxable income after deducting the state tax. Formulate this situation for determining the bonus, as a linear system. (4)

b) Apply the Gaussian elimination process to determine values of $\lambda$ for which the following linear system is consistent:

$$x - 3y + 4 = 0,\ 3x - 2y = \lambda,\ y = 6 - 2x.$$ (3)

c) Solve by substitution, the system you have obtained in 5(a). (3)

6. a) Give examples, with justification, of the following:

i) two non-zero, $3\times 3$ matrices $A$ and $B$, with $|A| = 0, |B| = \frac{5}{7}i$;

ii) two non-singular $2\times 2$ matrices $C$ and $D$, with $|C| = \sqrt{2}|D|$. (4)

b) Is Cramer's Rule applicable for solving the linear system below? If yes, apply it. Otherwise, alter the last equation in the system so that the solution can be obtained by applying the Rule.

$$x + y + z = \pi$$
$$-\pi x + \pi y + \sqrt{2}z = 0$$
$$\pi^2 x + \pi^2 y + 2z = 0.$$ (6)

7. a) Show that $1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{2(n-1)}$, for $n \in \mathbb{N}, n > 1$. (4)

b) Prove that $\frac{1}{2}(x+y+z) \leq \frac{x^2}{y+z} + \frac{y^2}{x+z} + \frac{z^2}{x+y}$, for $x, y, z > 0$. (4)

c) Let $x_i \in \mathbb{R}$ such that $0 < x_1 \leq x_2 \leq \ldots \leq x_n, n \geq 2$, and $\frac{1}{1+x_1} + \frac{1}{1+x_2} + \cdots + \frac{1}{1+x_n} = 1$. Then show that

$$\sqrt{x_1} + \sqrt{x_2} + \cdots + \sqrt{x_n} \geq (n-1)\left(\frac{1}{\sqrt{x_1}} + \cdots + \frac{1}{\sqrt{x_n}}\right).$$ (6)

d) Write an odd natural number as a sum of two integers $m_1$ and $m_2$ in a way that $m_1 m_2$ is maximum. (6)