MTE-03

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

MATHEMATICAL METHODS

(Valid from 1st January, 2023 to 31st December, 2023)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2023**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying all the blocks)

Course Code: MTE-03
Assignment Code: MTE-03/TMA/2023
Maximum Marks: 100

1. State whether the following statements are true or false giving reasons in support to your answers. (5×2=10)

   i) For the given vectors $\mathbf{a}, \mathbf{b}, \mathbf{c},$ if $\mathbf{b} = \mathbf{c},$ then $\mathbf{a} \times \mathbf{b} = \mathbf{c} \times \mathbf{a}$.

   ii) The function
   $$f(x) = \begin{cases} x^2 & \text{when } x \neq 1 \\ 2 & \text{when } x = 1 \end{cases}$$
   is discontinuous at $x = 1$.

   iii) The algebraic sum of deviations of 20 observations measured from 30 is 20. Then the mean of these observations is 30.

   iv) If $P(A \cap B) = \frac{1}{2}, P(\overline{A} \cap \overline{B}) = \frac{1}{2}$ and $2P(A) = P(B) = P,$ then $P = \frac{1}{3}$.

   v) The asymptote of the curve $x^3 + xy^2 - y^2 = 0$ parallel to y axis is $x = 1$.

2. a) Let $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be functions defined respectively by $f(x) = x^2 + 3x + 1,\ g(x) = 2x - 3.$ Find
   i) $f \circ g$
   ii) $\frac{f}{g}$
   iii) $f \circ f$ and
   iv) $f - g,$ if they exist and specify their domain also. (4)

   b) Evaluate $\int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx$. (3)

   c) The probabilities of A, B, C solving a problem are $\frac{1}{3}, \frac{2}{7},$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve it. (3)

3. a) For 5 observations of pairs (x, y) of variables x and $y,$ the following results are obtained: $\sum x = 15, \sum y = 25, \sum x^2 = 55, \sum y^2 = 135, \sum xy = 83.$ Find the two lines of regression. Also estimate the values of x and $y,$ if $y = 12$ and $x = 8$. (5)

b) Suppose a protein of mass m disintegrates into amino acids according to the formula $m = \frac{28}{t+2}$, where t indicates time. Find the average rate of reaction in the time interval $t = 0$ to $t = 2$. (3)

c) Find the term free of x in the Binomial expansion of $\left(4x - \frac{5}{x^2}\right)^6$. (2)

4. a) Calculate mode, Q_1, Q_2, Q_3 and quartile deviation for the following data: (5)

Marks	**No. of Students**
0-10	6
10-20	5
20-30	8
30-40	15
40-50	7
50-60	6
60-70	3

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. (3)

c) Evaluate $\lim_{x \to 4} \frac{x^2 - 16}{\sqrt{x^2 + 9 - 5}}$. (2)

5. a) Among 64 offsprings of a certain cross between guinea pigs, 34 were red, 10 were black and 20 were white. According to the genetic model these numbers should be in the ratio 9 : 3 : 4. Are the data consistent with the model at 5% level?
[The following values of χ^2 may be useful:
$\chi^2_{3,\,.05} = 7.81$
$\chi^2_{2,\,.05} = 5.99$
$\chi^2_{1,\,.05} = 3.84$] (4)

b) By eliminating a and b. Find the differential equation. Whose solution is given as: $y^2 = a(b - x)(b + x)$. (3)

c) If vector $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Find $|\mathbf{a} \times \mathbf{b}|$ and a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. (3)

6. a) If X is a normal variate with mean 30 and variance 25, find the probabilities that
i) $26 \le X \le 40$
ii) $X \ge 45$
(you may like to use the following values $\phi(.8) = .7881$, $\phi(2) = .9772$, $\phi(3) = .9987$, $\phi(1) = .8413$). (4)

b) Find four numbers forming a G.P. in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18. (3)

c) If the equation of the tangent at the point (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$, then find the values of a and b. (3)

7. a) Find the equation of the circle having the line joining the points (1, 2) and (3,4) as diameter. Also find its centre and radius. (3)

b) Solve the differential equation: $\frac{dy}{dx} = xy^3 - xy$. (4)

c) Find the asymptotes of the curve $y^2(x-1) - x^3 = 0$. (3)

8. a) Evaluate: $\int_0^1 x \tan^{-1} x \, dx$. (4)

b) Find the equation of the line parallel to the line $x = 4 - h,\ y = 2 + 3h,\ z = -4 + h$ and passing through $(3, 1, 5)$. (4)

c) Check the continuity at $x = 0$ of the function f where

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}.$$ (2)

9. a) 4 cards are drawn from a pack of cards. Find the probability that there is one card of each suit. (2)

b) Evaluate the following limit: $\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$. (3)

c) Find the mean and standard deviation for the following data: (5)

Marks	No. of Students
0-10	10
10-20	30
20-30	20
30-40	0
40-50	10
50-60	30

10. a) Solve: $(x + 2y^3)\, dy = y\, dx$. (4)

b) Do the vectors $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$, $b = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, and $\mathbf{c} = \mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ form a right angled triangle? (3)

c) Out of 24 bulbs in a shop, 4 bulbs are defective. If we randomly check two bulbs, then what is the probability that

i) both the bulbs are defective.

ii) neither of them is defective.

iii) one of them is defective? (3).