MTE-14

ASSIGNMENT BOOKLET
Bachelor's Degree Programme

MATHEMATICAL MODELLING

Valid from 1st January, 2022 to 31st December, 2022

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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THE PEOPLE'S UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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……………………………………………

COURSE CODE : …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE : ……………………….….. DATE :……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2022. If you have failed in this assignment or fail to submit it by December, 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-14
Assignment Code: MTE-14/TMA/2022
Maximum Marks: 100

1. a) Give a recent situation/incidence from the real world where you would like to formulate mathematical model to study the problem involved. State the type of modeling you will use for this problem giving reasons in support of your answer. Further, list at least 3 essentials and 3 non-essentials for this problem. (5)

   b) A particle is projected from the base of an inclined plane making an angle α with the horizontal. The initial velocity of project is making an angle β with the horizontal.

   i) Derive the equation to the trajectory.

   ii) Find the point at which the particle strikes the plane. (5)

2. a) A particle is moving under a central force along the path given by the equation $r = a \tan\theta$. Show that the radial acceleration is $\frac{k^2}{r^3}\left(3 + \frac{2a^2}{r^2}\right)$ where, $k = r^2 \frac{d\theta}{dt}$. (5)

   b) For the problem of finding the period of oscillation of a simple pendulum, obtain a formulation in the case when air resistance is proportional to the square of the velocity. Find a solution for this formulation. (5)

3. a) A raindrop starts falling from the clouds at a considerable height above the surface of the earth. During the fall, the raindrop experiences retardation due to air resistance, which is directly proportional to the instantaneous speed $v(t)$ of the drop. (5)

   i) Write the model equations

   ii) Is this system static or dynamic? Why?

   iii) Obtain an expression for the speed $v(t)$.

   iv) Discuss the behaviour of $v(t)$ as the time t changes.

   b) A stone is dropped vertically from a tower of height h. At the same time, another stone is thrown vertically upwards from the base of the tower with a velocity u. What is the minimum value of u so that the two stones will meet each other mid-air? (5)

4. a) A patient is given a dose Q mg/ml of a drug at regular interval of time t. The concentration C, of the drug in the blood has been shown experimentally to obey the law (6)

$$\frac{dC}{dt} = -k\, e^{C}$$

   i) If the first dose is administered at $t = 0$ hr, then find the concentration after T hr. have elapsed.

   ii) Assuming an instantaneous rise in concentration whenever the drug is administered, find the concentration after the second dose and T hr. have elapsed again.

   iii) Show that the limiting value R of the concentration for doses of Q mg/ml repeated at interval T hr. is given by the formula

$$R = -\ln \frac{kT}{1 - e^{-Q}}$$

b) Discuss the static stability and dynamic stability for the following demand and supply functions where we assume $k = 3$, $D_t = -0.2\,p_t + 80$, $S_t = 0.3\,p_t + 40$ (4)

5. a) Consider the population $x(t)$ of fish and $y(t)$ of sharks at any time t in a certain region of the pacific ocean. Sharks prey on fishes and their population would decrease if solved from the fish population. Let us assume further the following assumptions. (12)

1) The change in the shark and fish populations, in isolation, is respectively proportional to the present population of sharks and fish.

2) The number of sharks and fish caught by fishermen is directly proportional to the present population of the shark and fish population respectively. The fishing methods do not discriminate between sharks or fish.

3) The number of fish eaten by sharks is directly proportional to the product of the number of fish present and the number of sharks present.

4) The additional number of sharks surviving is directly proportional to the number of fish eaten.
Under assumptions 1) – 4).

i) Formulate the mathematical model for the given problem and write a system of differential equations governing it.

ii) Is the system of equations obtained in (i) above an autonomous system? Give reasons.

iii) Find the steady-state solution of the system in i) above. What does the solutions obtained correspond to?

iv) From the system of equations obtained in i) obtain the expression for $\frac{dy}{dx}$ and hence solve it under the condition $x(0) = x_0,\ y(0) = y_0$.

b) Consider a closed population of homogeneously mixing individuals with no removals. Suppose that $a(a > 0)$ numbers of infectives are introduced to a group of n susceptible at time $t = 0$ and infection spreads by the contact between the infectives and susceptible. (8)

i) Formulate the mathematical model for the given problem and write a differential equation governing it.

ii) Find the number of susceptible and infectives at any time t.

iii) Find the time when the rate of appearance of new infectives is maximum and also the density of susceptible at that time.

6. a) In a perfectly competitive market, the supply function $S(p)$ and demand function $D(p)$ are given as follows: (4)

$$S(p) = -\frac{p^2 + 17}{4};\ D(p) = -p + 5,\ \text{where p is the price:}$$

i) Find the equilibrium prices

ii) Using Walrasian stability condition, determine whether the equilibrium prices are stable.

b) Find the current density when oxygen diffuses through a red cell of thickness $2.5 \times 10^{-8}\,\text{cm}$, the two ends of which are maintain at a fixed concentration C_0. (3)

c) Let the slab represents a biological cell in a large bathing solution of solute with fixed (given) concentration of 13 mg. Then find the concentration distribution inside the cell at any given position $0 < x < h$ and time $t > 0$. (3)

7. a) Television sets for repair arrive at random at an average of 4 per day to a single repairman who takes an average of $1\frac{1}{2}$ hours to carry out each repair. It being assumed that the repair times have an exponential distribution. What is the average number of television sets in the workshop? What is the probability that an arriving set will find at least 3 sets in front of it? The repairman works for eight hours a day. (5)

b) Consider the cubic total cost function

$$C = 0.004q^3 - 0.8q^2 + 10q + 5$$

Assume that the price of q is 13 per unit. Find the output which yields maximum profit. (5)

8. a) Suppose that the previous forecast was 2083 and the actual value of the variable of interest for the last period was 1975 and the oldest value of interest was 1945. Using the moving average technique based upon the most recent four observations find new forecast for the next period. (5)

b) Let the utility function of a consumer be $U = q_1\sqrt{q_2}$. Let $p_1 = \text{Rs.}250$, $p_2 = \text{Rs.}400$ and that the consumers income for the period is Rs.15000. Obtain the quantities required by the consumer so that his utility function is maximized by consuming this combination. (5)

9. a) Let $P = (w_1, w_2)$ be a portfolio of two securities. Find the value of w_1 and w_2 in the following Situations (6)

i) $\rho_{12} = -1$ and P is risk-free.

ii) $\sigma_1 = \sigma_2$ and variance on P is minimum

iii) Variance on P is minimum and $\rho_{12} = -0.5$, $\sigma_1 = 2$ and $\sigma_2 = 3$.

b) There are N identical firms producing a particular commodity. The cost function for each firm producing q units is $q^3 + 2q^2 + 4q + 6$ units of money. Obtain the supply function for each firm. The demand function is $D(p) = N\left(\frac{22}{3} - p\right)$. Find the equilibrium price. (4)