MTE-07

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

ADVANCED CALCULUS

Valid from 1st January 2022 to 31st December 2022

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE FORMAT ABOVE STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of the very thin variety) for writing your answers.
3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2022. If you have failed in this assignment or fail to submit it by December, 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

ASSIGNMENT

Course Code: MTE-07
Assignment Code: MTE-07/TMA/2022
Maximum Marks: 100

1. State whether the following statements are true or false. Justify your answers in the form of a short proof or a counter-example: (10)

(a) If $S_1 = \{(x, y, z) \in \mathbf{R}^3 : x^2 + y^2 + z^2 \le 1\}$ and $S_2 = \{(x, y, z) \in \mathbf{R}^3 : |x| \le 1, |y| \le 1, |z| \le 1\}$, then $S_1 \subset S_2$.

(b) $\lim\limits_{x \to \infty} \dfrac{e^x}{e^x - 1}$ does not exist.

(c) $f_{xy}(0,0) \ne f_{yx}(0,0)$ for the function $f : \mathbf{R}^2 \to \mathbf{R}$ defined by:

$f(x, y) = \sin(e^x + e^y)$

(d) The following functions are not functionally dependent on $D = \{(x, y) \in \mathbf{R}^2 \,|\, x > 1, y > 0\}$:

$f(x, y) = \dfrac{2x + 2y}{2x}$ and $g(x, y) = \dfrac{2x + 2y}{2x}$.

(e) The function $f : \mathbf{R}^3 \to \mathbf{R}$ defined by $f(x, y, z) = |x + y + z|$ is integrable on $[0,1] \times [0,1] \times [0,1]$.

[Hint: Please note that only if you have checked the function is integrable, then only you can compute the integral.

2) (a) Calculate the double integral of the function $f : \mathbf{R}^2 \to \mathbf{R}$ defined by $f(x, y) = x + y$ over the region bounded by $x = 0, y = 4$ and $y = 2x$. (3)

(b) Let $f(x, y) = \begin{cases} \dfrac{x^3 - 8y^3}{x^2 + 4y^2}, & (x, y) \ne (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

Show that the function $f : \mathbf{R}^2 \to \mathbf{R}$ is continuous on $\mathbf{R}^2$. (5)

(c) Find the maximum possible domain and the corresponding range of the quotient function $\dfrac{f}{g}$, where $f : \mathbf{R}^2 \to \mathbf{R}$ are defined by $f(x, y) = 4xy$ and $g(x, y) = x^3 + y^3$. (2)

3) (a) If x, y, z, u and v are related by the equations $xy + yz + uv = 0$ and $x^2 + y^2 + z^2 + u^2 + v^2 = 0$ then compute $\dfrac{\partial u}{\partial y}$. (4)

(b) Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function defined by

$$f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & \text{otherwise}\end{cases}$$

Check whether or not f has a directional derivative at $(0,0)$ in the direction $\theta=\dfrac{\pi}{4}$. Deduce that the function f is not differentiable at the point $(0,0)$. (3)

(c) Under what condition on k does $\lim\limits_{x\to 0}\dfrac{kx\cos x-\sin x}{x^2\sin x}$ exist? Also find the limit when it exists. (3)

4. (a) Find the second Taylor polynomial of the function $f:\mathbf{R}^2\to\mathbf{R}$ defined by $f(x,y)=10+3x^2+2y^2$ at $(1,0)$. (4)

(b) Verify the implicit function theorem at the point $(2,2)$ for the function $f:\mathbf{R}^2\to\mathbf{R}$ defined by $f(x,y)=x^2-y^2$. (3)

[Hint: You need to check all the conditions of the theorem.]

(c) Let $e_1=(1,0,0)$. $e_2=(0,1,0)$ and $e_3=(0,0,1)$. Show that $x=e_1+2e_2$ and $y=e_2+e_3$ represent the points $(1,2,0)$ and $(0,1,1)$ respectively. Find the distance of the point $x+5y$ from the origin. (3)

5. (a) Using the method of Lagrange's multipliers, find the extreme points of the functions $f:\mathbf{R}^2\to\mathbf{R}$ defined by $f(x,y)=xy$, on the plane $x+y=1$. Further, check whether or not f has a local maximum at the extreme point. (4)

(b) Define the differentiability of a function $f:\mathbf{R}^2\to\mathbf{R}$ as a linear function. Use the definition to check whether the function $f:\mathbf{R}^2\to\mathbf{R}$ defined by $f(x,y)=x+y^2+5xy$ is differentiable at $(1,1)$. (4)

(c) Let $f:\mathbf{R}^2\to\mathbf{R}$ be a function defined by $f(x,y)=\tan\left(\dfrac{x^3-3y^3}{x^3+2y^3}\right)$. Show that

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=0.$$ (2)

6. (a) Evaluate the following integral by making the indicated change of variable

$$\iiint_W \frac{x+2y-z}{1+(y+3z)^2}\,dx\,dy\,dz$$

where $W: 0\le x+2y-z\le 3;$
$0\le y-z\le 2;$
$0<y+3z\le 1$

and

transformation: $u=x+2y-z$
$v=y-z$
$w=y+3z.$ (6)

(b) Find the work done by a force $F = (x^2, y^2)$ in moving a particle from the point $(1,1)$ to $(2,2)$ along the line segment from $(1,1)$ to $(2,1)$ followed by the line segment from $(2,1)$ to $(2,2)$. (4)

7. (a) Let $f:[0,2]\times[3,4] \to \mathbf{R}$ be defined by

$$f(x,y) = \begin{cases} 2, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

Show that $L(P,f) = 2$ and $U(P,f) = 4$, for any partition P of the rectangle $[0,2]\times[3,4]$. (3)

(b) Let $f(x,y) = x^2 - 9xy + 5y^2$. Find the directional derivative of f at $(1,2)$ in the direction $\theta = \dfrac{\pi}{6}$. (3)

(c) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$. (3)

8. (a) Find the second order partial derivatives of the function $f(x,y) = x^3 + x^2y^3 - 2y^2$ (3)

(b) Show that the following functions are functionally dependent on the domain $]0,\pi[\times\mathbf{R}$:

$$f(x,y) = e^y \sin^2 x, g(x,y) = y + 2\ln\sin x$$ (4)

(c) Evaluate the integral $\iint_D (x - 3y^2)dxdy$, where $D = \{(x,y): 0 \le x \le 2, 1 \le y \le 2\}$. (3)

9. (a) If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$ and $z = r^2s\sin t$, find the value of $\dfrac{\partial u}{\partial s}$, when $r = 2, s = 1, t = 0$. (6)

(b) Find the repeated limits of the following function at $(0,0)$ and check whether they are equal or not

$$f(x,y) = \frac{(y-3x)(2+x^2)}{(2y+x)(1+y^2)}$$

Further check whether the simultaneous limit exist. (4)

10. (a) Locate and classify the stationary points of the function

$$f(x,y) = x^2 + y^2 - 6xy + 6x + 3y - 4$$ (3)

(b) If $f(x,y,z) = x + y + z$ and $g(x) = 2x$, do $f\circ g$ and $g\circ f$ exist? Give reasons for your answer. (2)

(c) Find the minimum value of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. (5)