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MST-003

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

December, 2023

MST-003 : PROBABILITY THEORY

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining (Question Nos. 2 to 7).*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Use of formulae and statistical tables booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

P. T. O.

1. State whether the following statements are True or False. Give reasons in support of your answers :

$$2 \times 5 = 10$$

- (a) If A can hit the target in 4 out of 5 shots and B can hit the target in 3 out of 4 shots, then the probability that the target being hit when both try is $1/20$.
- (b) If X is a discrete random variable with

$$p(x) = \frac{x}{15} \quad \text{where } x = 1, 2, 3, 4, 5$$
$$= 0 \quad \text{otherwise.}$$

$$\text{Then } P(X = 1 \text{ or } 2) = 3/5.$$

- (c) If the mean and variance of $X \sim B(n, p)$ are 16 and 8, then n is 2.
- (d) If $X \sim N(5, 9)$ and $Y \sim N(3, 16)$ are two independent normal variates, then

$$(X - Y) \sim N(8, 7).$$

- (e) If A and B are two independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cup B)$ will be 0.55.

2. (a) In a bolt factory, machines A, B and C manufacture, respectively, 25, 35 and 40 percent of the total product. Of their output 5, 4 and 2 percent, respectively, are defective bolts. A bolt is randomly selected from a lot and is found to be defective. What is the probability that the bolt is selected was manufactured by (i) machine A, (ii) machine B, (iii) machine C ? 6
- (b) Three groups of employees have respectively 3 women and 1 man, 2 women and 2 men and 1 woman and 3 men. One employee is selected at random from each group. What is the probability that three selected consist of 1 woman and 2 men ? 2
- (c) State Bayes's theorem and total probability law. 2
3. (a) Let X be a random variable with pdf : 3

$$f(x) = C(1 - x^2) \text{ where } 0 < x < 1.$$

Then find (i) C, (ii) E(X), (iii) $P\left[\frac{1}{2} < X < \frac{3}{4}\right]$.

- (b) A discrete random variable X is defined as follows :

X	$P(X = x)$
0	K
1	$3K$
2	0.2
3	K
4	$2K + 0.1$

Find (i) the value of K and the probability distribution of X and (ii) $P(X > 2)$. 2

- (c) (i) Draw the graph of the distribution function given by : 2

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } 0 < x < 1 \\ 1 & \text{for } x \geq 1 \end{cases}$$

- (ii) Define probability mass function, probability density function and distribution function. 3

4. (a) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing :

- (i) at least 2 defectives

- (ii) exactly 2 defective items, and
- (iii) at most 2 defective items in a consignment of 1000 packets using Poisson distribution. 4
- (b) If a person is throwing stones at a target, what is the probability that his 10th throw is his 5th hit, if the probability of hitting the target at any trial is 0.5 ? 2
- (c) A panel of 7 Judges is to decide which of the 2 final contestants A and B will be declared the winner. A simple majority of the Judges will determine the winner. Assume that 4 of the Judges will vote for A and the rest will vote for B. If we randomly select 3 of the Judges and seek their verdict, what is the probability that a majority of them will favour A ? 4
5. (a) In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $r = 3$. If the powerplant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day ? 5

- (b) The marks obtained by the students in statistics in an examination are normally distributed with mean 150 and standard deviation 14.14. Find the probability that a student selected at random has secured a total of (i) 180 or above, and (ii) 135 or less. 5

6. (a) A die is of the shape of a regular tetrahedron whose faces have the numbers 111, 112, 121, 122. A_1 , A_2 , A_3 are, respectively, events that the first two, the last two and the extreme two digits are the same, when the die is tossed at random. Find whether or not the events A_1 , A_2 , A_3 are pairwise independent. 5

- (b) If : 3

$$f(x) = 2x \text{ when } 0 \leq x \leq 1$$

$$= 0 \text{ otherwise.}$$

find :

(i) $P\left(X < \frac{1}{2}\right)$

(ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$, and

(iii) $P\left(X > \frac{3}{4} \text{ given } X > \frac{1}{2}\right)$

(c) State and prove Lack of memory property of exponential distribution. 2

7. (a) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean 1.8. Find the probability that this computer will function for a month : (i) without breakdown, (ii) with only one breakdown and (iii) with at least one breakdown. 3

(b) The mileage of a car with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000 km. Find the probability that one of these tyres will last : 4

(i) at least 20,000 km, and

(ii) at most 30,000 km.

(c) Write conditions when binomial and Poisson distribution converges to Normal distribution. 3