No. of Printed Pages : 4 **MMT-007**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2023

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions out of remaining question nos.* ***2*** *to* ***7****.*

(iii) Use of scientific and non-programmable calculator is allowed.

1. State whether the following statements are true *or* false. Justify your answer with the help of a short proof or a counter-example. No marks will be awarded without justification : 5×2=10

   (i) Continuity of the function $f(x,y)=\sqrt{|y|}$ is not sufficient for the unique solution of the initial value problem $y'=\sqrt{|y|}$, $y(0)=0$ on the rectangle $|x|\leq 1, |y|\leq 1$.

P. T. O.

(ii) If inverse Laplace transform is denoted by L^{-1}, then :

$$L^{-1}\left[\ln\left(\frac{s+b}{s+a}\right)\right] = \frac{e^{-at} - e^{-bt}}{t}$$

(iii) The partial differential equation $u_{xx} + 4x\, u_{xy} + (1-y^2)\, u_{yy} = 0$ is elliptic p.d.e., inside the ellipse $4x^2 + y^2 < 1$.

(iv) For the boundary value problem :

$$y'''(x) = 0$$

$$y(0) = y(1) = 0$$

$$y'(0) + y'(1) = 0$$

Green's function exists.

(v) The approximate value of $y(0.6)$ for the initial value problem $y' = \sqrt{x+y}$, $y(0.4) = 0.41$ using the second order Runge-Kutta method is 0.61, where $h = 0.2$.

2. (a) Solve, in series, the differential equation : 6

$$x^2 y'' + 6xy' + (6 + x^2)y = 0$$

about $x = 0$.

(b) Express $f(x) = x^4 + 3x^3 + 4x^2 - x + 2$ in terms of Legendre polynomials. 4

3. (a) Find the Fourier transform of e^{-9x^2}. 4

(b) Find the solution of the heat conduction equation subject to the given initial and boundary conditions : 6

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1$$

$$u(x,0) = \sin(\pi x) \text{ for } 0 \le x \le 1$$

$$u(0, t) = 0 = u(1, t)$$

Using Laasonen method with $\lambda = \frac{1}{6}$ and $h = \frac{1}{3}$. Integrate for two levels.

4. (a) Use Fourier transforms to solve the boundary value problem : 5

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0$$

subject to the conditions :

(i) $u, \frac{\partial u}{\partial x} \to 0$ as $x \to \pm\infty$

(ii) $u(x, 0) = f(x)$

(b) Solve the initial value problem $y' = x^2 + y^2$, $y(0) = 1$, upto $x = 0.2$ using third order Taylor series method with $h = 0.1$. 5

5. Find the solution of $\nabla^2 u = 0$ in R subject to the boundary conditions : 10

$$u(x, y) = x^2 - y^2 \text{ on } x = 0,\ y = 0,\ y = 1;$$

$$u + \frac{\partial u}{\partial x} = x^2 + 2x - y^2 \text{ on } x = 1,$$

where R is the square $0 \le x \le 1$, $0 \le y \le 1$, using the five point formula. Use central difference approximation in the boundary condition. Assume uniform step length h = 1/2 along the axes.

6. Solve the boundary value problem : 10

$$y'' - 3y' + 2y = 2$$

with

$$y(0) - y'(0) = -1,$$

$$y(1) + y'(1) = 1$$

using the second order finite difference method with $h = \frac{1}{2}$.

7. (a) Using the generating function $J_n(x)$, prove that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$, for integer values of n. 5

(b) Evaluate : 5

$$\int_{-1}^{1} \frac{P_n(x)}{\sqrt{1 - 2xt + t^2}} dx,$$

where $P_n(x)$ is Legendre polynomial.