No. of Printed Pages : 6 **BME-001**

BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

02688

Term-End Examination

December, 2012

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks :* **70**

Note : *All questions are* ***compulsory****. Use of statistical tables and calculator is permitted.*

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1. Answer *any five* of the following : **5x4=20**

   (a) Evaluate any one of the following limits :

   (i) $\lim\limits_{x \to 0} \dfrac{x e^{x} - \log(1+x)}{x^{2}}$

   (ii) $\lim\limits_{x \to 1} x^{\frac{1}{1-x}}$

   (b) If $x^{y} = e^{x-y}$, find $\dfrac{dy}{dx}$

(c) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$;

show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

(d) Show that $v = \frac{A}{\gamma} + B$ is solution of the

differential equation $\frac{d^2 v}{d\gamma^2} + \frac{2}{\gamma}\frac{dv}{d\gamma} = 0.$

(e) Solve (any one of the following) :

(i) $(x^2 + y^2)\, dy = xy\, dx$

(ii) $\frac{dy}{dx} + \frac{y}{x} = \sin x^2.$

(f) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$; $w = \frac{xy}{z}$,

show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4.$

2. Answer *any four* of the following : **4x4=16**

(a) Calculate the total work done in moving a particle in a force field given by

$F = 3xy\hat{i} - 5z\hat{j} + 10x\,\hat{k}$ along the curve

$x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

(b) Show that $F=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Find also the work done in moving out an object in the field from $(1, -2, 1)$ to $(3, 1, 4)$.

(c) Verify the Divergence Theorem for the vector function

$F=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken

over the rectangular parallelopiped

$0 \leq x \leq a,\ 0 \leq y \leq b,\ 0 \leq z \leq c.$

(d) If $A = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$,

find $\nabla \times A$ (or curl A) at the point $(1, -1, 1)$.

(e) Find constants a, b, c so that

$V = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$

is irrotational.

(f) If $\phi = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5.$

find $\nabla^2\phi$.

3. Answer *any six* of the following : **6x3=18**

(a) If $A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$ show that $A^2 - 3A - 13\,I = 0$ where I and 0 are unit and zero (2×2) matrices respectively.

(c) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random

(i) 1 (ii) 0 (iii) at most 2

bolts will be defective.

(d) Assume that the probability of an individual coal miner being killed in a mine accident during a year is 1/2400. Use Poisson distribution, to calculate the probability that in a mine employing 200 miners, there will be at least one fatal accident in a year.

(e) A random sample of 10 boys had the following

IQ : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of a population mean IQ of 100 (at 5% level of significance) ?

(f) A certain number of articles manufactured in one batch were classified into three categories according to a particular characteristics, being less than 50, between 50 and 60 and greater than 60. If this characteristics is known to be normally distributed, determine the mean and standard deviation for the batch if 60%, 35% and 5% were found in these categories.

(b) Obtain the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 3 & 2 & 3 \end{bmatrix}$$

(c) Find the eigen values of the following matrix :

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

(d) Using Cayley-Hamilton theorem, find the inverse of $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

(e) Show that $A = \begin{bmatrix} 3 & 7-4\hat{i} & -2+5\hat{i} \\ 7+4\hat{i} & -2 & 3+\hat{i} \\ -2-5\hat{i} & 3-i & 4 \end{bmatrix}$

is a Hermitian matrix.

(f) Given $A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & -4 \end{bmatrix}$ and

$$B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \\ 5 & 2 & -1 \end{bmatrix}$$

Find where possible $A + B$, $A - B$, AB and BA, stating the reasons where the operations are not possible.

(g) Prove without expanding the determinant of

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$$

(h) Solve by Cramer's rule

$$2x - z = 1$$
$$2x + 4y - z = 1$$
$$x - 8y - 3z = -2.$$

4. Answer *any two* of the following : **4x4=16**

(a) An urn contain 10 balls of which three are black and seven are white. At each trial, a ball is selected at random, its colour is noted and it is replaced by two additional balls of the same colour.

What is the probability that a white ball is selected in the second trial ?

(b) A factory manufacturing televisions has four units A, B, C and D. The units A, B, C, D manufacture 15%, 20%, 30% and 35% of the total output respectively. It was found that out of their output 1%, 2%, 2% and 3% are defective. A television is chosen at random from the total output and found to be defective. What is the probability that it came from the unit D ?